New Solutions to Generalized Kadomtsev Petviashvili-Benjamin-Bona-Mahony and Generalized Zakharov Kuznetsov-Benjamin Bona Mahony-Joseph Egri Equation

M. S. AL-Amry[1]
Department of Mathematics
Aden, Aden University

S. A. AL-Damki[2]
Department of Mathematics
Faculty of Education Aden, Aden University

Abstract: In this paper, we present a new models of Kadomtsev-Petviashvili and Benjamin-Bona-Mahony (KP-BBM) equation, namely, a generalized of (KP-BBM) and Zakharov Kuznetsov-Benjamin Bona Mahony-Joseph Egri equation, namely, a generalized of (ZK-BBM-JE). We apply the sin-cosine method to solve them.

Exact travelling wave solutions are obtained and expressed in terms of hyperbolic functions and trigonometric functions.

Keywords: generalized of (KP-BBM) equation, generalized of (ZK-BBM-JE) equation, exact solutions and sin-cosine method .

Introduction: Any physical, biological and chemical phenomena can be modeled using nonlinear partial differential equations (NLPDEs). Many researchers have been interested in obtaining exact solutions of NLPDEs by using some methods as: the tanh method [5], the extended tanh-function method (ETM) [8], the simplest equation method [7], the integral bifurcation method [6], the extended mapping transformation method [21,1], the Backlund transformation of Riccati equation method (BTREM) [10,20], Hirota's direct method [2], ($G_{\circ}$/G ,1/G) expansion method [3], the mapping method [11], extended rational sinh-cosh and sin-cosine methods [12] and the extended generalized Riccati equation mapping method [4,9,13,14]. The present work is motivated by the desire to employ a sin-cosine method to nonlinear physical models [18,19], our approach is based mainly on a priori assumption that the solutions can be expressed in terms of sin or cosine functions.

Description of the Sin-Cosine Method

1. The wave variable $\xi = \mu(x - ct)$ carries a NLPDE in two independent variables

$$p(u, u_x, u_t, u_{xx}, u_{xt}, \dots) = 0. \quad (1)$$

to a nonlinear ordinary differential equation (NLODE)

$$Q(u, u', u'', \dots) = 0 , \quad (2)$$

Eq.(2) is then integrated as long as all terms contain derivatives where integration constants are neglected.

2. The solutions of many NLPDES can be expressed in the form

$$u(x,t) = \begin{cases} \{\lambda\cos^{\beta}(\mu\xi)\}, & |\xi| \le \frac{\pi}{2\mu} \\ 0, & \text{otherwise} \end{cases} \qquad (3)$$

or in the form

$$u(x,t) = \begin{cases} \{\lambda\sin^{\beta}(\mu\xi)\}, & |\xi| \le \frac{\pi}{\mu} \\ 0, & \text{otherwise} \end{cases} \qquad (4)$$

where λ, μ, and β are parameters that will be determined, μ and c are the wave number and the wave speed respectively, we then use

$$(u^n)' = -n\mu\beta\lambda^n cos^{n\beta-1}(\mu\xi)\sin(\mu\xi)$$
$$(u^n)'' = -n^2\mu^2\beta^2\lambda^n cos^{n\beta}(\mu\xi) + n\mu^2\lambda^n\beta(n\beta-1)cos^{n\beta-2}(\mu\xi) \qquad (5)$$

or

$$(u^n)' = -n\mu\beta\lambda^n sin^{n\beta-1}(\mu\xi)\cos(\mu\xi)$$
$$(u^n)'' = -n^2\mu^2\beta^2\lambda^n sin^{n\beta}(\mu\xi) + n\mu^2\lambda^n\beta(n\beta-1)sin^{n\beta-2}(\mu\xi)\ , \qquad (6)$$

3. Substituting Eq. (5) or Eq. (6) into the integrated NLODE gives a trigonometric equation of $\cos^{\beta}(\mu\xi)$ or $\sin^{\beta}(\mu\xi)$ terms. The parameters are λ, μ, and β then obtained by equating the exponents of each pair of cosine or sine, and by collecting all coefficients of the same power in $\cos^{k}(\mu\xi)$ or $\sin^{k}(\mu\xi)$, and set it equal to zero.

Exact Solutions for g(KP-BBM) Equation

In this section, we present our proposed equation, namely, a generalized Kadomtsev Petviashvili and Benjamin-Bona-Mahony equation

$$(v_t + v_x + a(v^m)_x - bv_{xxt})_x + cv_{yy} = 0, \qquad v = v(x,y,t), \quad (m\ integer) > 3, \qquad (7)$$

and denoted by g(KP-BBM) [16], where a, b, and c are arbitrary nonzero-constants.

We solve g(KP-BBM), by using the sin-cosine method.

Substituting $v(x,y,t) = v(\xi),\ \xi = (x + y - \omega t)$, in Eq.(7), we get

$$\left((1-\omega)v'(\xi) + b\omega v'''(\xi) + a\left(v^n(\xi)\right)'\right)' + cv''(\xi) = 0, \qquad m = n \qquad (8)$$

and integrating the resulting equation, we find

$$(1 + c - \omega)v(\xi) + b\omega v''(\xi) + av^n(\xi) = 0, \qquad (9)$$

Eq.(9) is nonlinear ordinary differential equation.

Now, we apply the sin-cosine method, to solve our equation. Consequently, we get the original solutions as the follows:

Substituting Eq.(3) or Eq.(4) in Eq.(9), using Eq.(5) or Eq.(6), we obtain

$$(1+c-\omega)\lambda cos^{\beta}(\mu\xi)+a\left(\lambda cos^{\beta}(\mu\xi)\right)^{n}$$
$$+b\omega\left(-\lambda\mu^{2}\beta^{2}\,cos^{\beta}(\mu\xi)+\mu^{2}\lambda\beta(\beta-1)cos^{\beta-2}(\mu\xi)\right)=0. \qquad (10)$$

Equating the exponents of the second and the last cosine functions, collecting the coefficients of each pair of cosine functions of the same exponent and setting it equal to zero, we obtain the following system of algebraic equations:

$$\beta-1\neq 0\,,\ n\beta=\beta-2\,,\ \omega b\lambda\mu^{2}\beta^{2}=(1+c-\omega)\lambda,$$
$$a\lambda^{n}=-\omega b\mu^{2}\lambda\beta(\beta-1). \qquad (11)$$

Solving this system leads to

$$\beta=\frac{-2}{n-1}\ ,\ \ \mu=\frac{n-1}{2}\sqrt{\frac{(1+c-\omega)}{b\omega}}\ \ ,\ \ b\omega\neq 0\,,$$
$$\lambda=\left(-\frac{(1+c-\omega)(n+1)}{2a}\right)^{\frac{1}{n-1}}\ . \qquad (12)$$

That can be easily obtained if we also use the sine method (4).

In view of the results (12), we get the following periodic solutions:

$$v(x,t)=\left(-\frac{(1+c-\omega)(n+1)}{2a}\,csc^{2}\left(\frac{n-1}{2}\sqrt{\frac{(1+c-\omega)}{b\omega}}\,(x+y-\omega t)\right)\right)^{\frac{1}{n-1}},\ \ |\mu\xi\,|<\pi$$

and

$$v(x,t)=\left(-\frac{(1+c-\omega)(n+1)}{2a}\,sec^{2}\left(\frac{n-1}{2}\sqrt{\frac{(1+c-\omega)}{b\omega}}\,(x+y-\omega t)\right)\right)^{\frac{1}{n-1}},\ \ |\mu\xi\,|<\frac{\pi}{2}$$

where $\frac{(1+c-\omega)}{b\omega}>0\,.$

However, for $\frac{(1+c-\omega)}{b\omega}<0$, we obtain the solitary patterns solutions

$$v(x,t)=\left(\frac{(1+c-\omega)(n+1)}{2a}\,csch^{2}\left(\frac{n-1}{2}\sqrt{\frac{(1+c-\omega)}{-b\omega}}\,(x+y-\omega t)\right)\right)^{\frac{1}{n-1}},$$

and

$$v(x,t)=\left(-\frac{(1+c-\omega)(n+1)}{2a}\,sech^{2}\left(\frac{n-1}{2}\sqrt{\frac{(1+c-\omega)}{-b\omega}}\,(x+y-\omega t)\right)\right)^{\frac{1}{n-1}}.$$

A Variant of the g(KP-BBM) Equation:

We next consider the nonlinear generalized Kadomtsev Petviashvili-Benjamin-Bona-Mahony equation with negative exponent

$$(v_{t}+v_{x}+a(v^{-n})_{x}-bv_{xxt})_{x}+cv_{yy}=0, v=v(x,y,t),\qquad n>1 \qquad (13)$$

Now, we apply the sin-cosine method, to solve our equation. Consequently, we get the original solutions as the follows:

substituting $v(x, y, t) = v(\xi),\ \xi = (x + y - \omega t)$, in Eq. (13), we get

$$\left((1-\omega)v'(\xi) + b\omega v'''(\xi) + a\left(v^{-n}(\xi)\right)'\right)' + cv''(\xi) = 0, \tag{14}$$

and integrating the resulting equation, we find

$$(1 + c - \omega)v(\xi) + b\omega v''(\xi) + av^{-n}(\xi) = 0, \tag{15}$$

Eq.(15) is nonlinear ordinary differential equation.

The cosine ansatz Eq.(3) takes Eq.(15) to

$$(1+\gamma-\omega)\lambda cos^{\beta}(\mu\xi) + \alpha\left(\lambda cos^{\beta}(\mu\xi)\right)^{-n} + b\omega(-\lambda\mu^2\beta^2 \cos^{\beta}(\mu\xi)$$
$$+\mu^2\lambda\beta(\beta-1)cos^{\beta-2}(\mu\xi)) = 0. \tag{16}$$

Equating the exponents of the second and the fourth cosine functions, and equating the coefficients of cosine functions of the same power as used before we get:

$$\beta - 1 \neq 0\ ,\quad -n\beta = \beta - 2,\quad b\omega\lambda\mu^2\beta^2 = (1 + c - \omega)\lambda,$$
$$a\lambda^{-n} = -b\omega\mu^2\lambda\beta(\beta - 1). \tag{17}$$

Solving this system leads to

$$\beta = \frac{2}{n+1}\ ,\quad \mu = \frac{n+1}{2}\sqrt{\frac{(1+c-\omega)}{b\omega}}\ , b\omega \neq 0\ ,\quad \lambda = \left(\frac{2a}{(1+c-\omega)(n-1)}\right)^{\frac{1}{n+1}}\ . \tag{18}$$

That can be easily obtained if we also use the sine method (4). In view of the results (18), the following periodic solutions:

$$v(x,t) = \left\{\left(\frac{2a}{(1+c-\omega)(n-1)}\ sin^2\left(\frac{n+1}{2}\sqrt{\frac{(1+c-\omega)}{b\omega}}\ (x+y-\omega t)\right)\right)^{\frac{1}{n+1}},\ |\mu\xi\ | \leq \pi\right.$$

and

$$v(x,t) = \left\{\left(\frac{2a}{(1+c-\omega)(n-1)}\ cos^2\left(\frac{n+1}{2}\sqrt{\frac{(1+c-\omega)}{b\omega}}\ (x+y-\omega t)\right)\right)^{\frac{1}{n+1}},\ |\mu\xi\ | \leq \frac{\pi}{2}\right.$$

where $\frac{(1+\gamma-\omega)}{b\omega} > 0.$

However, for $\frac{(1+c-\omega)}{b\omega} < 0$, we obtain the solitary patterns solutions

$$v(x,t) = \left\{\left(\frac{-2a}{(1+c-\omega)(n-1)}\ sinh^2\left(\frac{n+1}{2}\sqrt{\frac{(1+c-\omega)}{-b\omega}}\ (x+y-\omega t)\right)\right)^{\frac{1}{n+1}},\right.$$

and

$$v(x,t) = \left\{\left(\frac{2a}{(1+c-\omega)(n-1)}\ cosh^2\left(\frac{n+1}{2}\sqrt{\frac{(1+c-\omega)}{-b\omega}}\ (x+y-\omega t)\right)\right)^{\frac{1}{n+1}}.\right.$$

Exact Solutions for g(ZK-BBM-JE) Equation:

In this section, we consider generalized Zakharov Kuznetsov-Benjamin Bona Mahony - Joseph Egri equation

$$v_t + v_x + a(v^m)_x + b(v_{xt} + v_{yy} + v_{tt})_x = 0, \;\; v = v(x,y,t) \;\; (m\, integer) \neq \pm 1, \quad (19)$$

and denoted by g(ZK-BBM-JE) [15,17].

Now, we apply the sin-cosine method, to solve our equation.

Consequently, we get the original solutions as the follows:

substituting $v(x,y,t) = v(\xi),\ \xi = (x - \omega t)$, in Eq.(19) we get

$$(1-\omega)v'(\xi) + b(1-\omega+\omega^2)v'''(\xi) + a(v^n(\xi))' = 0\,, \qquad m = n \qquad (20)$$

Integrating once Eq.(20), we find

$$(1-\omega)v(\xi) + b(1-\omega+\omega^2)v''(\xi) + av^n(\xi) = 0\,, \qquad (21)$$

Eq.(21) is nonlinear ordinary differential equation.

The cosine ansatz Eq. (3) takes Eq.(21) to

$$(1-\omega)\lambda cos^{\beta}(\mu\xi) + a\left(\lambda cos^{\beta}(\mu\xi)\right)^{n} + b(1-\omega+\omega^2)(-\lambda\mu^2\beta^2\, cos^{\beta}(\mu\xi)$$
$$+\mu^2\lambda\beta(\beta-1)cos^{\beta-2}(\mu\xi)) = 0. \qquad (22)$$

Equating the exponents of the second and the fourth cosine functions, and equating the coefficients of cosine functions of the same power as used before we get:

$$\beta - 1 \neq 0, \;\; n\beta = \beta - 2, \;\; b(1-\omega+\omega^2)\lambda\mu^2\beta^2 = (1-\omega)\lambda\,,$$
$$a\lambda^{n-1} = -b(1-\omega+\omega^2)\mu^2\lambda\beta(\beta-1). \qquad (23)$$

Solving this system leads to

$$\beta = \frac{-2}{n-1},\;\; \mu = \frac{n-1}{2}\sqrt{\frac{(1-\omega)}{b(1-\omega+\omega^2)}}\;\; , b(1-\omega+\omega^2) \neq 0,$$
$$\lambda = \left(\frac{(\omega-1)(n+1)}{2a}\right)^{\frac{1}{n-1}}\,. \qquad (24)$$

That can be easily obtained if we also use the sine method (3).

In view of the results (24), the following periodic solutions:

$$v(x,t) = \left(\frac{(\omega-1)(n+1)}{2a}\, csc^2\left(\frac{n-1}{2}\sqrt{\frac{(1-\omega)}{b(1-\omega+\omega^2)}}\,(x+y-\omega t)\right)\right)^{\frac{1}{n-1}}, \;\; 0 < |\mu\xi| < \pi$$

and

$$v(x,t) = \left(\frac{(\omega-1)(n+1)}{2a}\, sec^2\left(\frac{n-1}{2}\sqrt{\frac{(1-\omega)}{b(1-\omega+\omega^2)}}\,(x+y-\omega t)\right)\right)^{\frac{1}{n-1}}, \;\; |\mu\xi| < \frac{\pi}{2}$$

where $\frac{(1-\omega)}{b(1-\omega+\omega^2)} > 0.$

However, for $\frac{(1-\omega)}{b(1-\omega+\omega^2)} < 0$, we obtain the solitary patterns solutions

$$v(x,t) = \left(-\frac{(\omega-1)(n+1)}{2a}\ csch^2 \left(\frac{n-1}{2} \sqrt{\frac{(1-\omega)}{-b(1-\omega+\omega^2)}}\ (x+y-\omega t) \right) \right)^{\frac{1}{n-1}},$$

and

$$v(x,t) = \left(\frac{(\omega-1)(n+1)}{2a}\ sech^2 \left(\frac{n-1}{2} \sqrt{\frac{(1-\omega)}{-b(1-\omega+\omega^2)}}\ (x+y-\omega t) \right) \right)^{\frac{1}{n-1}}.$$

A Variant of the g(ZK-BBM-JE) Equation:

We next consider the nonlinear generalized Zakharov Kuznetsov-Benjamin Bona Mahony-Joseph Egri equation with negative exponent

$$v_t + v_x + a(v^{-n})_x + b(v_{xt} + v_{yy} + v_{tt})_x = 0, \quad v = v(x,y,t), \qquad n > 1 \tag{25}$$

Now, we apply the sin-cosine method, to solve our equation.

Consequently, we get the original solutions as the follows:

substituting $v(x,y,t) = v(\xi),\ \xi = (x - \omega t)$, in Eq.(25), we get

$$(1-\omega)v'(\xi) + b(1-\omega+\omega^2)v'''(\xi) + a(v^{-n}(\xi))' = 0\,, \tag{26}$$

Integrating once Eq.(26), we find

$$(1-\omega)v(\xi) + b(1-\omega+\omega^2)v''(\xi) + av^{-n}(\xi) = 0\,, \tag{27}$$

Eq.(27) is nonlinear ordinary differential equation.

The cosine ansatz Eq.(3) takes Eq.(27) to

$$(1-\omega)\lambda cos^{\beta}(\mu\xi) + a\left(\lambda cos^{\beta}(\mu\xi)\right)^{-n} + b(1-\omega+\omega^2)(-\lambda\mu^2\beta^2\ cos^{\beta}(\mu\xi)$$
$$+ \mu^2\lambda\beta(\beta-1)cos^{\beta-2}(\mu\xi) = 0. \tag{28}$$

Equating the exponents of the second and the fourth cosine functions, and equating the coefficients of cosine functions of the same power as used before we get:

$$-n\beta = \beta - 2\,,\ \ b(1-\omega+\omega^2)\lambda\mu^2\beta^2 = (1-\omega)\lambda \quad \beta - 1 \neq 0\,,$$
$$a\lambda^{-n} = -b(1-\omega+\omega^2)\mu^2\lambda\beta(\beta-1). \tag{29}$$

Solving this system leads to

$$\beta = \frac{2}{n+1}\ ,\ \mu = \frac{n+1}{2}\sqrt{\frac{(1-\omega)}{b(1-\omega+\omega^2)}}\ ,\ \omega \neq 0\,,$$
$$\lambda = \left(\frac{2a}{(\omega-1)(1-n)}\right)^{\frac{1}{n+1}}. \tag{30}$$

That can be easily obtained if we also use the sine method (4). In view of the results (30), the following periodic solutions:

$$v(x,t) = \begin{cases} \left(\frac{2a}{(\omega-1)(1-n)}\ sin^2 \left(\frac{n+1}{2} \sqrt{\frac{(1-\omega)}{b(1-\omega+\omega^2)}}\ (x+y-\omega t) \right) \right)^{\frac{1}{n+1}}, & |\mu\xi\ | \leq \pi \end{cases}$$

and

$$v(x,t) = \left\{\left(\frac{2a}{(\omega-1)(1-n)}\cos^2\left(\frac{n+1}{2}\sqrt{\frac{(1-\omega)}{b(1-\omega+\omega^2)}}\,(x+y-\omega t)\right)\right)^{\frac{1}{n+1}}, \quad |\mu\xi\;| \le \frac{\pi}{2}\right.$$

where $\frac{(1-\omega)}{b(1-\omega+\omega^2)} > 0.$

However, for $\frac{(1-\omega)}{b(1-\omega+\omega^2)} < 0$, we obtain the solitary patterns solutions

$$v(x,t) = \left(\frac{-2a}{(\omega-1)(1-n)}\sinh^2\left(\frac{n+1}{2}\sqrt{\frac{(1-\omega)}{-b(1-\omega+\omega^2)}}\,(x+y-\omega t)\right)\right)^{\frac{1}{n+1}}$$

and

$$v(x,t) = \left(\frac{2a}{(\omega-1)(1-n)}\cosh^2\left(\frac{n+1}{2}\sqrt{\frac{(1-\omega)}{-b(1-\omega+\omega^2)}}\,(x+y-\omega t)\right)\right)^{\frac{1}{n+1}}.$$

Conclusions:

In this paper, we generalized Zakharov Kuznetsov-Benjamin Bona Mahony-Joseph Egri equation g(ZK-BBM-JE) and Kadomtsev Petviashvili- Benjamin-Bona-Mahony equation g(KP-BBM). Then, we give its exact solutions by applying the sin-cosine method. The results show that this methods are a powerful mathematical tool for obtaining exact solutions for our equations.

It is also a promising methods to solve other nonlinear partial differential equations.

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حلول جديدة لمعادلة كادومتسيف بيتفياشفيلي- بنيامين- بونا- ماهوني المعممة ومعادلة زخاروف كوزنيتسوف- بنيامين بونا ماهوني- جوزيف إيجري المعممة

محمد سـالم العـمـري
قسم الرياضيات كلية التربية عدن, جامعة عدن

سلمى عبدالله الدعمكي
قسم الرياضيات كلية التربية عدن, جامعة عدن

الملخص: في هذه الورقة, نقدم نماذج لمعادلة كادومتسيف - بيتفياشفيلي وبنيامين - بونا - ماهوني, وهي معادلة معممة لمعادلة (KP-BBM) ومعادلة زاخاروف كوزنيتسوف - بنيامين بونا ماهوني - جوزيف إيجرى, وهي معادلة معممة لمعادلة (ZK-BBM-JE). ونطبق طريقة الجيب وجيب التمام لحلها.
نحصل على حلول دقيقة للموجات المتنقلة ونعبر عنها بالدوال الزائدية والدوال المثلثية.

الكلمات المفتاحية: معادلة معممة لمعادلة (KP- BBM), معادلة معممة لمعادلة (ZK- BBM-JE), حلول دقيقة وطريقة الجيب وجيب التمام.