JEF/Journal of Education Faculties
Volume 18, Issue (1), 2024

مجلة كليات التربية - جامعة عدن
المجلد 18، العدد (1)، 2024 م

Study of Generalized Inverse Trigonometric Functions of Fourier Series Based on Floor Function

Anter Ali AL-saiyad Ali
Dept. of Math., Faculty of Education
Aden University, Yemen

Jehad Fadhel Ali AL-Dderbi
Dept. of Math., Faculty of Education
Laheg University, Yemen
E. mail: jhadaldrby144@gmail.com

Abstract. This paper is devoted for the study of certain generalized the inverse trigonometric functions in real numbers. We study on inverse trigonometric functions and periodic functions by their Fourier series. At the end of this paper, we got new identicals that represents a generalization of the inverse trigonometric functions.

Keywords: Floor Function, Fourier Series, Inverse Trigonometric Functions, Periodic Functions.

1. Introduction: The Bernoulli polynomials, which play an important role in Analytic numbers Theory, are usually defined by means of the generating function

$$\frac{te^{tx}}{e^t-1} = \sum_{k=0}^{\infty} B_k(x)\frac{t^k}{k!} .$$

The polynomial $B_k(x)$ is monic and has degree k. for example $B_0(x) = 1$, $B_1(x) = x - \frac{1}{2}$.

In 1890, Humitz found the Fourier expansions **(see[4],[7])**

$$B_{2k}(x) = \frac{2(-1)^{k-1}(2k)!}{(2\pi)^{2k}}\sum_{k=1}^{\infty}\frac{cos(2\pi nx)}{n^{2k}} \quad , x \in [0,1[\, , \;\; k \geq 1, \tag{1.1}$$

$$B_{2k+1}(x) = \frac{2(-1)^{k-1}(2k+1)!}{(2\pi)^{2k+1}}\sum_{k=1}^{\infty}\frac{sin(2\pi nx)}{n^{2k+1}} \quad , x \in [0,1[\, , \;\; k \geq 1. \tag{1.2}$$

The Fourier expansions (1.1) and (1.2) are actually valid for $0 \leq x < 1$, and the convergence is absolute and uniform on $[0,1]$, except for $B_0(x)$ and $B_1(x)$.

So we can construct the periodic extension of $B_k(x)$ on $[0,1]$ to R by taking fractional parts $\{x\}$ and using $B_k(\{x\})$ instead of $B_k(x)$, these $[0,1]$-periodic extensions are continuous (see [7],[11]). Then (1.1) and (1.2) for $k \geq 1$ becone

$$B_{2k}(\{x\}) = \frac{2(-1)^{k-1}(2k)!}{(2\pi)^{2k}}\sum_{k=1}^{\infty}\frac{cos(2\pi nx)}{n^{2k}} \quad , x \in R \, , \quad . \tag{1.3}$$

$$B_{2k+1}(\{x\}) = \frac{2(-1)^{k-1}(2k+1)!}{(2\pi)^{2k+1}}\sum_{k=1}^{\infty}\frac{sin(2\pi nx)}{n^{2k+1}} \quad , x \in R \, . \tag{1.4}$$

We give new theorem is the main pillar of this paper (see the third section for details). Also using Bernoulli polynomials where, we take k=0 in equation (1.4) **(see[6])** in order to obtain results.

Finally, we got new identity, it is very important, in an applications for finding the summations, which are difficult to find the partial summation of them.

Now, we introduce some preliminaries that are very important in this paper.

2. Preliminaries

Definition 2.1 (see [2]). A series of the form

$$\frac{a_0}{2}+\sum_{n=1}^{\infty}\left(a_n cos\left(\frac{n\pi x}{L}\right)+b_n sin\left(\frac{n\pi x}{L}\right)\right). \tag{2.1}$$

is called a trigonometric series.

Definition 2.2. (see [2]) If the trigonometric series in (2.1) convergent uniform to the function f in [-L,L], under series of the form:

$$f(x)\sim\frac{a_0}{2}+\sum_{\mathrm{n}=1}^{\infty}\left(a_n\cos\left(\frac{n\pi x}{L}\right)+b_n\sin\left(\frac{n\pi x}{L}\right)\right).$$

Then $f(L)=f(-L)$ and the coefficients is

$$a_0=\frac{1}{L}\int_{-L}^{L}f(x)dx,$$

$$a_n=\frac{1}{L}\int_{-L}^{L}f(x)cos\left(\frac{n\pi x}{L}\right)dx,$$

And

$$b_n=\frac{1}{L}\int_{-L}^{L}f(x)sin\left(\frac{n\pi x}{L}\right)dx.$$

Definition 2.3. **(see** [9]). A function $f(x)$ of one variable x is periodic with T, if T> 0 and $f(x\pm T)=f(x)$.

Definition 2.4. (see [12],[13],[14],[15])The floor function of real number x is define by symbol $\lfloor x\rfloor$ that satisfies $\lfloor x\rfloor\le x<\lfloor x\rfloor+1$; the fraction part of x denoted by symbol$\{x\}$ That satisfies $x=\lfloor x\rfloor+\{x\}$; the ceiling function of x is denoted by symbol $\{x\}$ that fits $x\le\lceil x\rceil<x+1$.

Theorem 2.1. (Euler's summation formula.[1]). Let f be Piecewise smooth on the interval $[u,v]$, where $0<u<v$, then

$$\sum_{u<n\le v}f(n)=\int_u^v f(t)dt+\int_u^v(t-\lfloor t\rfloor)f'(t)dt+f(v)(\lfloor v\rfloor-v)-f(u)(\lfloor u\rfloor-u).$$

Theorem 2.2. (Weierstrass M-test)[3]. Let $\sum_{k=1}^{\infty}f_k(x)$ is a function series and there exist numerical series $\sum_{k=1}^{\infty}a_k$ be a convergent, such that $a_k\ge 0,\ \forall k\in Z$. If $|f_k(x)|\le a_k$, for all x∈ X, $\forall k\in Z$, then the function series $\sum_{k=1}^{\infty}f_k(x)$ be a convergent uniform on X.

Theorem 2.3. (Dirichlet's Test[9],[3])). Let $\{f_k\}$ and $\{g_k\}$ is two function sequences in X such that.

i) There is m> 0 and $|\sum_{k=1}^{n}f_k(x)|\le m,\ \forall m\ge 1$, x ∈ X,

ii) $g_{k+1}(x)\le g_k(x),\forall k,\forall x\in X$,

iii) $\{g_k\} \to 0$ uniformly on X.

then $\sum_{k=1}^{\infty} f_k g_k$ is uniformly convergent in X.

Now , we introduce some relations, it is very important in this paper **(see[10]).**

$$\sin^{-1} x + \cos^{-1} x = \frac{\pi}{2}, x \in [-1,1]. \quad (2.2)$$

$$\tan^{-1} x + \cot^{-1} x = \frac{\pi}{2}, \quad x \in R. \quad (2.3)$$

$$\csc^{-1} x + \sec^{-1} x = \frac{\pi}{2}, \text{ for } |x| \geq 1. \quad (2.4)$$

$$\sin^{-1} x + \sec^{-1}\left(\frac{1}{x}\right) = \frac{\pi}{2}, x > 0. \quad (2.5)$$

$$\cos^{-1} x + \csc^{-1}\left(\frac{1}{x}\right) = \frac{\pi}{2}, x > 0. \quad (2.6)$$

$$\tan^{-1} x + \tan^{-1}\left(\frac{1}{x}\right) = \frac{\pi}{2}, x > 0. \quad (2.7)$$

3. Generalized Inverse Trigonometric Functions

Now, we use Bernoulli polynomials in **(1.4)**, in order to prove the following theorem.

Theorem 3.1. Let the Inverse Trigonometric Functions be an integrable in R, then the follows hold:

i) $$sin^{-1}(cosx) = (-1)^{\left\lfloor \frac{x}{\pi} \right\rfloor}\left(\pi \left\lfloor \frac{x}{\pi} \right\rfloor - x + \frac{\pi}{2}\right) \text{ ,where } x \in R . \quad (3.1)$$

ii) $$tan^{-1}(cotx) = \pi \left\lfloor \frac{x}{\pi} \right\rfloor - x + \frac{\pi}{2} \quad \text{, where } x \neq \pi k, k \in Z.\ x \in R. \quad (3.2)$$

Where $\lfloor . \rfloor$ is Floor Function.

Proof

We prove this theorem by Fourier Series.

i) Put $f(x) = \sin^{-1}(\cos x)$, such that 2L=2π. Then, used **Definition 2.1.** We obtain

$a_0 = \frac{1}{\pi}\int_{-\pi}^{\pi} sin^{-1}(cosx)dx$, since a_0 is even, then observe that

$$a_0 = \frac{2}{\pi}\int_0^{\pi} sin^{-1}(cosx)dx ,$$

and

$$a_0 = \frac{2}{\pi}\int_0^{\pi} sin^{-1}\left(sin\left(\frac{\pi}{2} - x\right)\right)dx = \frac{2}{\pi}\int_0^{\pi}\left(\frac{\pi}{2} - x\right)dx = 0 .$$

$$a_n = \frac{2}{\pi}\int_0^{\pi} sin^{-1}(cosx)\cos(nx)\, dx = \frac{2}{\pi}\int_0^{\pi}\left(\frac{\pi}{2} - x\right)\cos(nx)\, dx ,$$

integration by Parts, we obtain

$a_n = \frac{2(1-(-1)^n)}{n^2\pi}$. Since b_n is odd, then $b_n = 0$.

Using Fourier Series in **Definition 2.1** is:

$$sin^{-1}(cosx) = \frac{4}{\pi}\sum_{n=1}^{\infty}\frac{cos(2n-1)x}{(2n-1)^2} . \quad (3.3)$$

The above is convergent uniform by **Theorem .2.2**. **(Weierstrass M-test)**

And, put $f(x) = (-1)^{\left\lfloor \frac{x}{\pi} \right\rfloor}$, since $f(x)$ is $2\pi -$ periodic, then by Fourier series are:

$$a_0 = \frac{1}{\pi}\int_0^{2\pi}(-1)^{\left\lfloor \frac{x}{\pi} \right\rfloor}dx = \frac{1}{\pi}\int_0^{\pi} dx + \frac{1}{\pi}\int_{\pi}^{2\pi} -dx = 0 .$$

$$a_n = \frac{1}{\pi}\int_0^{2\pi}(-1)^{\left\lfloor \frac{x}{\pi} \right\rfloor}\cos(nx)dx = \frac{1}{\pi}\int_0^{\pi} \cos(nx)dx + \frac{1}{\pi}\int_{\pi}^{2\pi} -\cos(nx)dx = 0 ,$$

and

$$b_n = \frac{1}{\pi}\int_0^{2\pi}(-1)^{\left\lfloor \frac{x}{\pi} \right\rfloor}sin(nx)dx = \frac{1}{\pi}\int_0^{\pi} sin(nx)dx + \frac{1}{\pi}\int_{\pi}^{2\pi} -sin(nx)dx = \frac{2(1-(-1)^n)}{n\pi} .$$

Hence

$$(-1)^{\left\lfloor \frac{x}{\pi} \right\rfloor} = \frac{4}{\pi}\sum_{n=1}^{\infty}\frac{sin(2n-1)x}{(2n-1)} . \qquad (3.4)$$

Using integral from 0 to x, $x \in R$. We obtain that

$$\int_0^x(-1)^{\left\lfloor \frac{t}{\pi} \right\rfloor} dt = \frac{4}{\pi}\sum_{n=1}^{\infty}\frac{1}{(2n-1)^2} - \frac{4}{\pi}\sum_{n=1}^{\infty}\frac{cos(2n-1)x}{(2n-1)^2}, \text{ since}$$

$$\sum_{n=1}^{\infty}\frac{1}{(2n-1)^2} = \frac{3}{4}\zeta(2) , \quad \zeta(2) = \frac{\pi^2}{6} , \text{(see [5])}.$$

We compare from equation **(3.3)**,

$$\int_0^x(-1)^{\left\lfloor \frac{t}{\pi} \right\rfloor} dt = \frac{\pi}{2} - sin^{-1}(cosx) . \qquad (3.5)$$

We evaluate the above integral

$$\int_0^x(-1)^{\left\lfloor \frac{t}{\pi} \right\rfloor} dt \quad put\ y = \frac{t}{\pi}. \text{ Then}$$

$$\int_0^x(-1)^{\left\lfloor \frac{t}{\pi} \right\rfloor} dt = \pi\int_0^{\left\lfloor \frac{x}{\pi} \right\rfloor}(-1)^{\lfloor y \rfloor}dy + \pi\int_{\left\lfloor \frac{x}{\pi} \right\rfloor}^{\frac{x}{\pi}}(-1)^{\lfloor y \rfloor}dy ,$$

by **Theorem 2.1. Euler's summation formula,** we get

$$\int_0^x(-1)^{\left\lfloor \frac{t}{\pi} \right\rfloor}dt = \pi\sum_{n=0}^{\left\lfloor \frac{x}{\pi} \right\rfloor - 1}(-1)^n + \pi(-1)^{\left\lfloor \frac{x}{\pi} \right\rfloor}\left(\frac{x}{\pi} - \left\lfloor \frac{x}{\pi} \right\rfloor\right)$$

$$= \frac{\pi}{2}\left(1 - (-1)^{\left\lfloor \frac{x}{\pi} \right\rfloor}\right) + (-1)^{\left\lfloor \frac{x}{\pi} \right\rfloor}\left(\frac{x}{\pi} - \left\lfloor \frac{x}{\pi} \right\rfloor\right),$$

hence

$$\int_0^x(-1)^{\left\lfloor \frac{t}{\pi} \right\rfloor}dt = \frac{\pi}{2} + (-1)^{\left\lfloor \frac{x}{\pi} \right\rfloor}\left(x - \pi\left\lfloor \frac{x}{\pi} \right\rfloor - \frac{\pi}{2}\right). \qquad (3.6)$$

From equation **(3.5)** and **(3.6),** we obtain on **(3.1),** which complete the proof.

iii) From equation (1.4), put k=0,we obtain

$$B_1(\{x\}) = -\frac{1}{\pi}\sum_{k=1}^{\infty}\frac{sin(2\pi kx)}{k} , x \in R.$$

After that, we find $B_1(\{x\})$ by Fourier series , we put $f(x) = \lfloor x \rfloor - x + \frac{1}{2}$

be a one-periodic. Then

$$a_0 = 2\int_0^1\left(\lfloor x \rfloor - x + \frac{1}{2}\right)dx = 0 .$$

And

$$a_n = 2\int_0^1\left(\lfloor x \rfloor - x + \frac{1}{2}\right)\cos(2\pi nx)dx = 2\int_0^1\left(\frac{1}{2} - x\right)\cos(2\pi nx)\,dx = 0 .$$

$$b_n = 2\int_0^1 \left(\lfloor x \rfloor - x + \frac{1}{2}\right)\sin(2\pi nx)\,dx = 2\int_0^1 \left(\frac{1}{2} - x\right)\sin(2\pi nx)\,dx$$

$$= \int_0^1 \sin(2\pi nx)\,dx - 2\int_0^1 x\sin(2\pi nx)\,dx = \frac{-1}{n\pi} x\cos(2\pi nx)]_0^1 = \frac{-1}{n\pi}.$$

Therefore, by **Definition 2.1.**we obtain that

$$f(x) = \frac{1}{\pi}\sum_{n=1}^{\infty}\frac{\sin(2n\pi x)}{n}. \qquad (3.7)$$

The above equation be uniformly convergent by **Theorem 2.3.(Dirichlet's Test**).

Then $B_1(\{x\}) = x - \lfloor x \rfloor - \frac{1}{2}$.

And we put $g(x) = tan^{-1}(cot\pi x)$ be a one−periodic. Then

$$a_0 = 2\int_0^1 tan^{-1}(cot\pi x)dx = 2\int_0^1 \left(\frac{\pi}{2} - \pi x\right)dx = 0.$$

And

$$a_n = 2\int_0^1 tan^{-1}(cot\pi x)\cos(2n\pi x)\,dx = \int_0^1 (\pi - 2\pi x)\cos(2n\pi x)\,dx,$$

Integration by parts ,we get.

$$a_n = \left[\frac{\pi - 2\pi x}{2n\pi} sin(2n\pi x)\right]_0^1 - \left[\frac{cos(2n\pi x)}{2n^2\pi}\right]_0^1 = 0.$$

And

$$b_n = 2\int_0^1 tan^{-1}(cot\pi x)sin(2\pi nx)\,dx = 2\int_0^1 \left(\frac{\pi}{2} - \pi x\right)\sin(2n\pi x)\,dx,$$

$$= -2\pi\left[\frac{-x cos(2n\pi x)}{2n\pi}\right]_0^1 = \frac{1}{n}.$$

Hence $tan^{-1}(cot\pi x)$ is:

$$tan^{-1}(cot\pi x) = \sum_{n=1}^{\infty}\frac{\sin(2n\pi x)}{n}. \qquad (3.8)$$

From equation (3.7) and (3.8), we obtain

$$\lfloor x \rfloor - x + \frac{1}{2} = \frac{1}{\pi} tan^{-1}(cot\pi x).$$

Put $y = \pi x$, which complete the proof.

Corollary 3.1. Let the Inverse Trigonometric function be an integrable in R, x ∈ R, then the following is satisfied:

i) $sin^{-1}(sinx) = (-1)^{\left\lfloor \frac{1}{2} - \frac{x}{\pi} \right\rfloor}\left(x + \pi\left\lfloor \frac{1}{2} - \frac{x}{\pi} \right\rfloor\right).$ (3.9)

ii) $cos^{-1}(cosx) = \frac{\pi}{2} - (-1)^{\left\lfloor \frac{x}{\pi} \right\rfloor}\left(\pi\left\lfloor \frac{x}{\pi} \right\rfloor - x + \frac{\pi}{2}\right).$ (3.10)

iii) $tan^{-1}(tanx) = x - \pi\left\lfloor \frac{1}{2} + \frac{x}{\pi} \right\rfloor, x \neq (2k+1)\frac{\pi}{2}, k \in Z.$ (3.11)

iv) $cot^{-1}(cotx) = x - \pi\left\lfloor \frac{x}{\pi} \right\rfloor, x \neq k\pi, k \in Z.$ (3.12)

v) $sec^{-1}(secx) = \frac{\pi}{2} - (-1)^{\left\lfloor \frac{x}{\pi} \right\rfloor}\left(\pi\left\lfloor \frac{x}{\pi} \right\rfloor - x + \frac{\pi}{2}\right), x \neq (2k+1)\frac{\pi}{2}, k \in N.$ (3.13)

vi) $csc^{-1}(cscx) = (-1)^{\left\lfloor \frac{1}{2} - \frac{x}{\pi} \right\rfloor}\left(x - \pi\left\lfloor \frac{1}{2} - \frac{x}{\pi} \right\rfloor\right), x \neq k\pi, k \in N.$ (3.14)

Proof.

i) put $x=\frac{\pi}{2}-x$ in identical (3.1), we have $sin\left(\frac{\pi}{2}-x\right)=cosx$, which complete the proof.

ii) Instead of x, take cosx in identity (2.2), we obtain on (3.10).

iii) Put $x=\frac{x}{2}+x$, in identity **(3.2)** and we have $tan\left(\frac{\pi}{2}+x\right)=-cotx$, the identity was to be proven.

(vi), (v) and (iv) are same proof identity (i) and (ii).

Lemma 3.1. Let $x\in R-Z$. Then the following identity hold:

$$(-1)^{\lfloor x\rfloor}=4\left\lfloor\frac{x}{2}\right\rfloor-2\lfloor x\rfloor+1. \tag{3.15}$$

proof:

suppose that $f(x)=4\left\lfloor\frac{x}{2}\right\rfloor-2\lfloor x\rfloor$ be period function to 2-eriodic. Thus

$$a_0=\int_0^2\left(4\left\lfloor\frac{x}{2}\right\rfloor-2\lfloor x\rfloor\right)dx=\int_1^2(-2)dx=-2\,.$$

And

$$a_n=\int_0^2\left(4\left\lfloor\frac{x}{2}\right\rfloor-2\lfloor x\rfloor\right)\cos(n\pi x\,)dx=\int_1^2(-2)\cos(n\pi x)\,dx=0\,.$$

$$b_n=\int_0^2\left(4\left\lfloor\frac{x}{2}\right\rfloor-2\lfloor x\rfloor\right)sin(n\pi x)dx=-2\int_1^2 sin(n\pi x)dx=\frac{2(1-(-1)^n)}{n\pi}.$$

Then, by **Definition2.1**, we obtain

$$4\left\lfloor\frac{x}{2}\right\rfloor-2\lfloor x\rfloor+1=-1+\frac{4}{\pi}\sum_{n=1}^{\infty}\frac{sin(2n-1)\pi x}{2n-1}\,. \tag{3.16}$$

The above equation is uniformly converges by **Theorem 2.3.(Dirichlet's Test).**

Now we put $f(x)=(-1)^{\lfloor x\rfloor}$ is 2-perodic , then by Fourier series:

$$a_0=\int_0^2(-1)^{\lfloor x\rfloor}dx=\int_0^1 dx-\int_1^2 dx=0.$$

And

$$a_n=\int_0^2(-1)^{\lfloor x\rfloor}cos(n\pi x)dx=\int_0^1 cos(n\pi x)dx-\int_1^2 cos(n\pi x)dx=0.$$

$$b_n=\int_0^2(-1)^{\lfloor x\rfloor}sin(n\pi x)dx=\int_0^1 sin(n\pi x)dx-\int_1^2 sin(n\pi x)dx$$

$$=\left[-\frac{cos(n\pi x)}{n\pi}\right]_0^1+\left[\frac{cos(n\pi x)}{n\pi}\right]_1^2=\frac{2(1-(-1)^n)}{n\pi}.$$

Thus

$$(-1)^{\lfloor x\rfloor}=\frac{4}{\pi}\sum_{n=1}^{\infty}\frac{sin(2n-1)\pi x}{2n-1}. \tag{3.17}$$

Then, from equation (3.16) and (3.17), we obtain (3.15).

Now, we apply at the above Lemma in the following example.

Example 3.1. Prove that

$$\sum_{n=1}^{\infty}\frac{(-1)^{\lfloor 2^n x\rfloor}}{2^n}=1+2(\lfloor x\rfloor-x)\,,\ \text{where }\ x\in R\,.$$

Proof.

Using identity (3.15), we obtain that

$$\sum_{n=1}^{\infty}\frac{(-1)^{\lfloor 2^n x\rfloor}}{2^n}=\sum_{n=1}^{\infty}\frac{4\lfloor 2^{n-1}x\rfloor-2\lfloor 2^n x\rfloor+1}{2^n}=\sum_{n=1}^{\infty}\frac{1}{2^n}+2\lim_{n\to\infty}\sum_{n=1}^{n}\left(\frac{\lfloor 2^{n-1}x\rfloor}{2^{n-1}}-\frac{\lfloor 2^n x\rfloor}{2^n}\right)$$

$$= \frac{\frac{1}{2}}{1-\frac{1}{2}} + 2\left[\left(\lfloor x\rfloor - \frac{\lfloor 2x\rfloor}{2}\right) + \left(\frac{\lfloor 2x\rfloor}{2} - \frac{\lfloor 3x\rfloor}{3}\right) + \cdots + \left(\frac{\lfloor 2^{n-1}x\rfloor}{2^{n-1}} - \frac{\lfloor 2^{n}x\rfloor}{2^{n}}\right)\right]$$

$$= 1 + 2\lim_{n\to\infty}\left(\lfloor x\rfloor - \frac{\lfloor 2^{n}x\rfloor}{2^{n}}\right) = 1 + 2(\lfloor x\rfloor - x).$$

References

[1] **Apostol, T.**M. .(2000). Introduction to Analytic Number Theory. Second Edition. *Undergraduate texts in mathematics*.PP:353.

[2] **Asher, K**. (2013). Fourier Series And Fourier Transform. *J. of mathematics*.4(6): 73-76.

[3] **Braian, S.TH and Judith, B.B and Andrew, M.B.** (2008). Real Analysis**.** Second Edition ,www.classicalrealanalysis.com.PP 1014.

[4] **Cheon, S.R and Jihee, Y and Yu, S.J**.(2014). Fourier Series of the Periodic Bernoulli and Euler Functions. http://dx.doi.org//10.1155/2014/856491.

[5] **Chunli, L and Wenchang, C.**(2022). Improper Integrals Involving Powers of Inverse Trigonometric and Hyperbolic Functions. *J. Mathematics MDPI*, https://doi.org/10.330/ math10162980.

[6] **Francisco, J. M. T and Eder, C. G**.(2016). The Poisson Summation Formula for Functions of Bounded Variation. *Journal of Mathematical Sciences*. 18(3): 90-96.

[7] **Gyong, W.J and Taekyun, K and Dae, S.K and Toufik, M**. (2017). Fourier series of functions related to Bernoulli polynomials. https://www .researchgate .net/ publication /313694261.

[8] **Jackie, N and Peggy, A.** (2006). Introduction Trigonometric Functions, First Edition. *University of Sydney NSW* .

[9] **Javier C. L.** (2016). Convergence of Fourier series. PhD Thesis, *University del pals vasco*, PP: 65.

[10] **Melham, R.S and Shannon, A.G.** ,(2007). Inverse Trigonometric and Hyperbolic Summation Formulas Involving Generalized Fibonacci numbers. First Edition .*University of Technology, Sydney*.

[11] **Navas, L. M and Ruiz, F. J and Varona, J.J**.(2020). The Mobius inversion formula for Fourier series applied to Bernoulli and Euler polynomials. In Journal of Approximation Theory: https://www .researchgate .net/ publication /220162531.

[12] **Wang, X.** (2018). Some inequalities on T3 tree. Advances in Pure Mathematics, 8(8), 711-719.

[13] **Wang, X.** (2018). T3 tree and its traits in understanding integers. *Advances in* Pure *Mathematics*, 8(5), 494-507.

[14] **Wang, X.** (2019). Brief summary of frequently-used properties of the floor function: Updated 2018. *IOSR Journal of Mathematics*, 15(1), 30–33.

[15] **Wang, X.(**2020).Frequently-Used Properties of the Floor Function. *in international Journal of Applied Physics and Mathematics*, 135-142.

دراسة حول تعميم الدوال المثلثية العكسية من متسلسلات فورية بناءً على دالة الأرض

عنتر علي الصياد
أستاذ- قسم الرياضيات- كلية التربية
جامعة عدن- اليمن

جهاد فضل علي الدربي
معيد- قسم الرياضيات- كلية التربية
جامعة لحج-اليمن

الملخص: ترتكز هذه الورقة على نظرية وحيده تعبر عن بعض الدوال المثلثية العكسية بدلالة دالة الصحيح. ندرس الدوال المثلثية العكسية والدوال الدورية من خلال متسلسلات فورية. في نهاية هذه الورقة، حصلنا على متطابقات جديدة تمثل تعميم للدوال المثلثية العكسية.

الكلمات المفتاحية: دالة الأرض، متسلسلات فورية، الدوال الدورية، الدوال المثلثية العكسية.