A Decompositions of Continuity in Ideal Topological Spaces

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Abstract: The intention of this work is to study the concepts of strong $\alpha^* - I -$ continuity and strong $\alpha^* - I -$ open (resp. closed) mappings in ideal topological spaces, obtain several characterizations and some properties of these mappings and investigate its relationship with other types of mappings. Also, we introduce strong $\alpha^* - I -$ separation axioms in ideal topological spaces and study some their characterize.

Keywords: Ideal topological spaces, strong $\alpha^* - I -$ continuous**,** strong $\alpha^* - I -$ open (closed) mapping, strong $\alpha^* - I -$ separation axioms.

1- Introduction and Preliminaries.

Quite recently, Hatir and Noiri [10] established the concept of $\alpha - I -$ continuous functions and utilized it to derive a decomposition of continuity. Açıkgöz, Noiri and Yüksel [1] established numerous characterizations of $\alpha - I -$ continuous functions and to introduce and obtain the features of $\alpha - I -$ open functions in ideal topological spaces. Dontchev [7] used ideals to investigate Hausdorff spaces.

Arenas, Dontchev and Puertas [5] introduced separation axioms in ideal topological spaces by connecting an open or closed set with a member of the ideal. These are referred to as T_1 spaces. Hatir and Noiri [11] proposed and investigated the concept of semi$-I-$ Hausdrorff spaces. Throughout this paper $cl(A)$ and $int(A)$ denote the closure and the interior of A, respectively. Let (X, τ) be a topological space and I an ideal of subsets of X. An ideal is defined as a nonempty collection I of subsets of X satisfying the following two conditions [13]:

(1) If $A \in I$ and $B \subset A$, then $B \in I$.

(2) If $A \in I$ and $B \in I$, then $A \cup B \in I$.

An ideal topological space is a topological space (X, τ) with an ideal I on X and is denoted by (X, τ, I). For a subset $A \subset X$, $A^*(I, \tau) = \{x \in X: U \cap A \notin I$, for each neighborhood $U \in \tau(X)\}$ is called the local function of A with respect to I and τ [14]. We simply write A^* instead of $A^*(I)$ in case there is no chance for confusion. For every ideal topological space (X, τ, I), there exists a topology $\tau^*(I)$, finer than τ, generated by $\beta(I, \tau) = U - I: U \in \tau$ and $I \in I$, but in general $\beta(I, \tau)$ is not always atopology, additionally, $cl^*(A) = A \cup A^*$ defines a Kuratowski closure operator for $\tau^*(I)$ [12].

In the form of Definition 1.1, we refer to the results reported in [2 – 4, 8 – 10].

Definition 1.1. A subset A of an ideal topological space (X, τ, I) is called:

(1) $S.S^* - I -$ open if $A \subset cl^*(int^*(A))$,

(2) $S.P^* - I -$ open if $A \subset int^*(cl^*(A))$,

(3) $\alpha - I -$ open if $A \subset int(cl^*(int(A)))$,

(4) $\beta^* - I -$ open if $A \subset cl(int^*(cl(A)))$,

(5) $b - I -$ open if $A \subset cl^*(int(A)) \cup int(cl^*(A))$,

(6) strong $\alpha^* - I -$ open if $A \subset int^*(cl^*(int^*(A)))$.

We mention the following results in the form of Definition 1.2 among the results reported in [6, 8 – 10].

Definition 1.2. A mapping $f: (X, \tau, I) \to (Y, \sigma)$ is called:

(1) $\alpha - I -$ continuous if $f^{-1}(V)$ is $\alpha - I -$ open in X, $\forall V \in \sigma$.

(2) $S.S^* - I -$ continuous if $f^{-1}(V)$ is $S.S^* - I -$ open set in X, $\forall V \in \sigma$.

(3) $S.P^* - I -$ continuous if $f^{-1}(V)$ is $S.P^* - I -$ open in X, $\forall V \in \sigma$.

(4) $\beta^* - I -$ continuous if $f^{-1}(V)$ is $\beta^* - I -$ open in X, $\forall V \in \sigma$.

(5) $b - I -$ continuous if $f^{-1}(V)$ is $b - I -$ open in X, $\forall V \in \sigma$.

2- Strong $\alpha^* - I -$ Continuous Mappings

Definition 2.1. A mapping $f: (X, \tau, I) \to (Y, \sigma)$ is called $S.\alpha^* - I -$ continuous if $f^{-1}(V)$ is a $S.\alpha^* - I -$ open set in (X, τ, I), $\forall V \epsilon \sigma$.

Example 2.2. Let $X = Y = \{a, b, c, d\}$, $\tau = \{\phi, X, \{a\}, \{a, b\}, \{a, b, c\}\}$, $\sigma = \{\phi, Y, \{a\}, \{b, d\}, \{a, b, d\}\}$ and $I = \{\phi, \{a\}\}$. If $f: (X, \tau, I) \to (Y, \sigma)$ defined by: $f(a) = a$, $f(b) = d$, $f(c) = b$, $f(d) = c$, then $f^{-1}(Y) = X$, $f^{-1}(\phi) = \phi$ and $f^{-1}(V)$ is $S.\alpha^* - I -$ open $\forall V \in \sigma$. Hence f is S.$\alpha^* - I -$ continuous.

The following diagram which holds for a mapping $f: (X, \tau, I) \to (Y, \sigma)$.

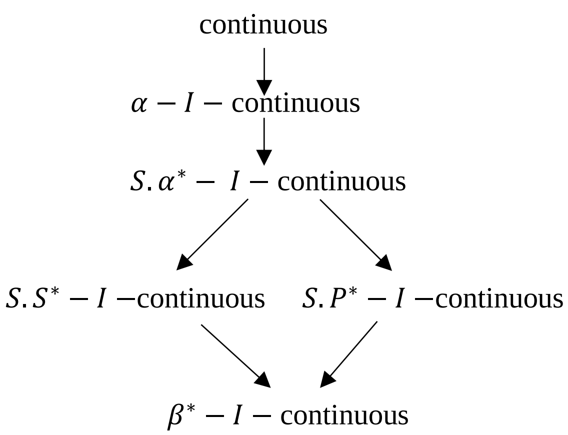

Fig. 1. The implication between some generalizations of continuous mapping.

Remark 2.3. The converse of these implications in Fig.1 are not true in general as shown in the following examples.

Example 2.4. From Example 2.2 f is $S.\alpha^* - I -$ continuous while f is not $\alpha - I -$ continuous because $f^{-1}(\{b,d\})$ is $S.\alpha^* - I -$ open, but $f^{-1}(\{b,d\})$ is not $\alpha - I -$ open.

Example 2.5. Let $X = Y = \{a,b,c,d\}$, $\tau = \{\phi, X, \{c\}, \{a,b,d\}\}$, $I = \{\phi, \{a\}\}$, $\sigma = \{\phi, Y, \{a\}, \{b\}, \{a,b\}\}$ and $f: (X,\tau,I) \to (Y,\sigma)$ defined by: $f(a) = c$, $f(b) = a$, $f(c) = d$, $f(d) = b$, then f is $S.P^* - I -$ continuous, but it is not $S.\alpha^* - I -$ continuous because $f^{-1}(\{a\})$ is $S.P^* - I -$ open, while $f^{-1}(\{a\})$ is not $S.\alpha^* - I -$ open.

Example 2.6. Let $X = Y = \{a,b,c\}$, $\tau = \{\phi, X, \{a\}, \{c\}, \{a,c\}\}$, $I = \{\phi, \{b\}\}$, $\sigma = \{\phi, Y, \{a\}, \{b,c\}\}$ and $f: (X,\tau,I) \to (Y,\sigma)$ defined by: $f(a) = a$, $f(b) = c$, $f(c) = b$. We notice that f is $S.S^* - I -$ continuous, but it is not $S.\alpha^* - I -$ continuous because $f^{-1}(\{b,c\})$ is $S.S^* - I -$ open, but $f^{-1}(\{b,c\})$ is not $S.\alpha^* - I -$ open.

Example 2.7. If $X = Y = \{a,b,c,d\}$, $\tau = \{\phi, X, \{a\}, \{a,b\}, \{a,b,c\}\}$, $I = \{\phi, \{a\}\}$, $\sigma = \{\phi, Y, \{c\}, \{b,c\}\}$ and $f: (X,\tau,I) \to (Y,\sigma)$ defined by: $f(a) = c$, $f(b) = a$, $f(c) = b$, $f(d) = d$, then f is $\beta^* - I -$ continuous but it not $S.\alpha^* - I -$ continuous because $f^{-1}(\{b,c\})$ is $\beta^* - I -$ open, while $f^{-1}(\{b,c\})$is not $S.\alpha^* - I -$ open.

Remark 2.8. $S.\alpha^* - I -$ continuous and $b - I -$ continuous are independent notions Examples (2.9, 2.10).

Example 2.9. Let $X = Y = \{a,b,c,d\}$, $\tau = \{\phi, X, \{a\}, \{a,b\}, \{a,b,c\}\}$, $I = \{\phi, \{a\}\}$, $\sigma = \{\phi, Y, \{c\}, \{a,c\}\}$ and $f: (X,\tau,I) \to (Y,\sigma)$ defined by: $f(a) = a$, $f(b) = c$, $f(c) = d$, $f(d) = b$. Then we get that f is $S.\alpha^* - I -$ continuous but not $b - I -$ continuous because $f^{-1}(\{c\})$ is$S.\alpha^* - I -$ open, while $f^{-1}(\{c\})$ is not $b - I -$ open.

Example 2.10. Let $X = Y = \{a,b,c\}$, $\tau = \{\phi, X, \{a\}, \{c\}, \{a,c\}\}$, $I = \{\phi, \{b\}\}$, $\sigma = \{\phi, Y, \{a\}, \{a,c\}\}$ and $f: (X,\tau,I) \to (Y,\sigma)$ defined by: $f(a) = a$, $f(b) = c$, $f(c) = b$. Then we obtain f is $b - I -$ continuous, while f is not $S.\alpha^* - I -$ continuous because $f^{-1}(\{a,c\})$ is $b - I -$open but $f^{-1}(\{a,c\})$ is not $S.\alpha^* - I -$ open.

Theorem 2.11 presents the relationship between $S.\alpha^* - I -$ continuous, $S.\alpha^* - I -$ closure and $S.\alpha^* - I -$interior

Theorem 2.11. Let $f: (X,\tau,I) \to (Y,\sigma)$ be a mapping. The following statements are equivalent:

(1) f is $S.\alpha^* - I -$ continuous,

(2) $\forall\, x \in X$ and $\forall\, V \in \sigma$ and $f(x) \in V$, $\exists\, S.\alpha^* - I -$ open set $W \subset X$ such that $x \in W$, $f(W) \subset V$.

(3) $f^{-1}(V)$ is $S.\alpha^* - I -$ closed, $\forall$ closed set $V \subset Y$.

(4) $f(S.\alpha^* Icl(A)) \subset cl(f(A))$, $\forall A \subset X$.

(5) $S.\alpha^* Icl(f^{-1}(V)) \subset f^{-1}(cl(V))$, $\forall V \subset Y$.

(6) $f^{-1}(int(V)) \subset S.\alpha^* Iint(f^{-1}(V))$, $\forall V \subset Y$.

Proof. (1) ⇒ (2) Let $x \in X$, $V \in \sigma$ and $f(x) \in V$. Put $W = f^{-1}(V)$, then by

Definition 2.1 W is $S.\alpha^* - I -$ open such that $x \in W$ and $f(W) \subset V$.

(2) ⇒ (3) Let V is a closed set of Y. Put $E = Y - V$, then $E \in \sigma$. Let $x \in f^{-1}(E)$, then by (2) we have $f(W) \subset E$. Thus,

$$x \in W \subset int^*(cl^*(int^*(W))) \subset int^*(cl^*(int^*(f^{-1}(E))))$$

hence $f^{-1}(E) \subset int^*(cl^*(int^*(f^{-1}(E))))$. This shows that $f^{-1}(E)$ is $S.\alpha^* - I -$ open. Hence

$$f^{-1}(V) = X - f^{-1}(Y - V) = X - f^{-1}(E).$$

This implies that $f^{-1}(V)$ is $S.\alpha^*$ - I-closed.

(3) ⇒ (4) Let $A \subset X$, then $f(A) \subset Y$ and because $f(A) \subset cl(f(A))$, then $A \subset f^{-1}(f(A)) \subset f^{-1}(cl(f(A)))$.

This implies that

$$S.\alpha^*Icl(A) \subset S.\alpha^*Icl(f^{-1}(cl(f(A)))) = f^{-1}(cl(f(A)))$$

and therefore,

$$f(S.\alpha^*Icl(A) \subset f(f^{-1}(cl(f(A)) \subset cl(f(A))$$

Hence $f(S.\alpha^*Icl(A)) \subset cl(f(A))$.

(4) ⇒ (5) Let $V \subset Y$, then $f^{-1}(V) \subset X$ and by (4) we get

$$f(S.\alpha^*Icl(f^{-1}(V))) \subset cl(f(f^{-1}(V))) \subset cl(V),$$ therefore,

$$f^{-1}(f(S.\alpha^*Icl(f^{-1}(V)))) \subset f^{-1}(cl(V)).$$

Hence

$$S.\alpha^*Icl(f^{-1}(V)) \subset f^{-1}(f(S.\alpha^*Icl(f^{-1}(V)))) \subset f^{-1}(cl(V)).$$

(5) ⇒ (6) Let $V \subset Y$, then $Y - V \subset Y$. Now by (5) we have $S.\alpha^*Icl(f^{-1}(Y - V)) \subset f^{-1}(cl(Y - V))$, Therefore $X - S.\alpha^*Iint(f^{-1}(V)) \subset X - f^{-1}(int(V))$. Hence $f^{-1}(int(V)) \subset S.\alpha^*Iint(f^{-1}(V))$.

(6) ⇒ (1) Let $V \in \sigma$, therefore

$$f^{-1}(V) = f^{-1}(int(V)) \subset S.\alpha^*Iint(f^{-1}(V)).$$

Hence $f^{-1}(V)$ is $S.\alpha^* - I -$ open $\forall V \in \sigma$. This shows that f is $S.\alpha^* - I -$ continuous.

The following theorems explain some of the characteristics of $S.\alpha^* - I -$ continuous.

Theorem 2.12. Let $f: (X, \tau, I) \to (Y, \sigma)$ be mapping. Then the following statements are equivalent:

(1) f is $S.\alpha^* - I -$ continuous.

(2) $cl^*(int^*(cl^*(f^{-1}(V)))) \subset f^{-1}(cl(V))$, $\forall V \subset Y$.

(3) $f(cl^*(int^*(cl^*(A)))) \subset cl(f(A))$, $\forall A \subset X$.

Proof. (i) ⇒ (ii) Since f is a $S.\alpha^* - I -$ continuous, $cl(V)$ is closed, then by Theorem 2.11 (3) $f^{-1}(cl(V))$ is $S.\alpha^* - I -$ closed $\forall V \subset Y$ and we have

$$cl^*(int^*(cl^*(f^{-1}(V)))) \subset cl^*(int^*(cl^*(f^{-1}(cl(V))))) \subset f^{-1}(cl(V)).$$

Hence

$$cl^*(int^*(cl^*(f^{-1}(V)))) \subset f^{-1}(cl(V)).$$

(2) ⇒ (3) Let $A \subset X$, then $f(A) \subset Y$ and by (2) we have

$$cl^*(int^*(cl^*(f^{-1}(f(A))))) \subset f^{-1}(cl(f(A))).$$ Therefore,

$$cl^*(int^*(cl^*(A))) \subset cl^*(int^*(cl^*(f^{-1}(f(A)))))$$

$\subset f^{-1}(cl(f(A)))$.

Hence

$f(cl^*(int^*(cl^*(A)))) \subset f(f^{-1}(cl(f(A))))$

$\subset cl(f(A))$.

(3) ⇒ (1) Let $V \in \sigma$, then $f^{-1}(Y - V) \subset X$ and by (3)

$f(cl^*(int^*(cl^*(f^{-1}(Y - V)))))$

$\subset cl(f(f^{-1}(Y - V)))$

$\subset cl(Y - V) = Y - V$.

Therefore,

$f(X - int^*(cl^*(int^*(f^{-1}(V))))) \subset Y - V$.

Hence

$X - int^*(cl^*(int^*(f^{-1}(V))))$

$\subset f^{-1}(f(X - int^*(cl^*(int^*(f^{-1}(V))))))$

$\subset f^{-1}(Y - V)$

$= X - f^{-1}(V)$.

Thus, $f^{-1}(V) \subset int^*(cl^*(int^*(f^{-1}(V))))$.

Hence $f^{-1}(V)$ is $S.\alpha^* - I -$ open. This shows that f is $S.\alpha^* - I -$ continuous

Theorem 2.13. Let $f: (X, \tau, I) \to (Y, \sigma)$ be a bijective mapping. Then f is $S.\alpha^* - I -$ continuous if and only if $int(f(A)) \subset f(S.\alpha^* Iint(A)), \forall A \subset X$.

Proof. Let f be a $S.\alpha^* - I -$ continuous, $\forall A \subset X$, then $f^{-1}(int(f(A)))$ is $S.\alpha^* - I -$ open. Because f is bijective and by Theorem 2.11 (6) we have

$int(f(A)) = f(f^{-1}(int(f(A))))$

$\subset f(S.\alpha^* Iint(f^{-1}(f(A))))$

$= f(S.\alpha^* Iint(A))$.

Hence $int(f(A)) \subset f(S.\alpha^* Iint(A))$.

Conversely, let $V \in \sigma$, then $V = int(V)$ and by hypothesis we obtain

$V = int(V)$

$= int(f(f^{-1}(V)))$

$\subset f(S.\alpha^* Iint(f^{-1}(V)))$

hence $V \subset f(S.\alpha^* Iint(f^{-1}(V)))$ this implies that

$f^{-1}(V) \subset f^{-1}(f(S.\alpha^* Iint(f^{-1}(V))))$

$= S.\alpha^* Iint(f^{-1}(V))$

therefore, $f^{-1}(V) \subset S.\alpha^* Iint(f^{-1}(V))$.

Hence $f^{-1}(V)$ is $S.\alpha^* - I -$ open. This shows that f is $S.\alpha^* - I -$ continuous.

Corollary 2.14. Let $f: (X, \tau, I) \to (Y, \sigma)$ be a bijective mapping. Then f is $S.\alpha^* - I -$ continuous if and only if $int(f(A)) \subset f(int^*(cl^*(int^*(A)))), \forall A \subset X$.

Proof. Let f be bijective $S.\alpha^* - I -$ continuous, then by Theorem 2.13 we get

$int(f(A)) \subset f(S.\alpha^* Iint(A))$

$\subset f(int^*(cl^*(int^*(S.\alpha^* Iint(A)))))$

$\subset f(int^*(cl^*(int^*(A))))$.

Conversely, let $V \in \sigma$, $f^{-1}(V) \subset X$. Since f is bijective and by hypothesis we Obtain

$V = int(V)$

$= int(f(f^{-1}(V)))$

$\subset f(int^*(cl^*(int^*(f^{-1}(V)))))$.

Therefore,

$f^{-1}(V)$

$\subset f^{-1}(f(int^*(cl^*(int^*(f^{-1}(V))))))$

$= int^*(cl^*(int^*(f^{-1}(V))))$.

Hence $f^{-1}(V)$ is $S.\alpha^* - I -$ open. This shows that f is $S.\alpha^* - I -$ continuous.

Remark 2.15. If $f: (X, \tau, I) \to (Y, \sigma, J)$ and $g: (Y, \sigma, J) \to (Z, \mu)$ are $S.\alpha^* - I -$ continuous mappings, then the composition $g \circ f: (X, \tau, I) \to (Z, \mu)$ may not be a $S.\alpha^* - I -$ continuous mapping. This can be shown by the following example.

Example 2.16. Let $X = Y = Z = \{a, b, c, d\}$, $\tau = \{\phi, X, \{c\}, \{a, b, d\}\}$, $I = \{\phi, \{a\}\}$ be an ideal on X, $\sigma = \{\phi, Y, \{a\}, \{b, c\}, \{a, b, c\}\}$, $J = \{\phi, \{a\}, \{d\}, \{a, d\}\}$ be an ideal on Y and $\mu = \{\phi, Z, \{a\}, \{a, b, d\}\}$. If $f: (X, \tau, I) \to (Y, \sigma, J)$ and $g: (Y, \sigma, J) \to (Z, \mu)$ defined as following: $f(a) = d$, $f(b) = b$, $f(c) = a$, $f(d) = c$, $g(a) = a$, $g(b) = c$, $g(c) = b$, $g(d) = d$, then g and f are $S.\alpha^* - I -$ continuous but $g \circ f: (X, \tau, I) \to (Z, \mu)$ is not $S.\alpha^* - I -$ continuous because $(g \circ f)^{-1}(\{a, b, d\}) = \{a, c, d\}$ is not $S.\alpha^* - I -$ open.

Definition 2.18. A mapping $f: (X, \tau, I) \to (Y, \sigma, J)$ is called $S.\alpha^* - I -$ irresolute if $f^{-1}(V)$ is a $S.\alpha^* - I -$ open set in (X, τ, I) for every $S.\alpha^* - I -$ open set $V \subset Y$.

Theorem 2.19. Let $f: (X, \tau, I) \to (Y, \sigma, J)$ be a mapping. Then the following statements are equivalent:

(1) f is a $S.\alpha^* - I -$ irresolute,

(2) $\forall x \in X$ and each $S.\alpha^* - I -$ open V in Y such that $f(x) \in V$, there exists A is $S.\alpha^* - I -$ open in X such that $x \in A$ and $f(A) \subset V$,

(3) $f^{-1}(V)$ is a $S.\alpha^* - I -$ closed set, for each $S.\alpha^* - I -$ closed set Vin X.

Proof. (1) ⇒ (2) Let $x \in X$, V is $S.\alpha^* - I -$ open in Y and $f(x) \in V$. Put $A = f^{-1}(V)$, then by Definition 2.18 A is $S.\alpha^* - I -$ open in X such that $x \in A$ and $f(A) \subset V$.

(2) ⇒ (3) Let V is $S.\alpha^* - I -$ closed of Y. Put $E = Y - V$, then E is $S.\alpha^* - I -$ open in Y. Let $x \in f^{-1}(E)$, then by (2) we have $f(A) \subset E$. Thus,

$$x \in A \subset int^*(cl^*(int^*(A)))$$
$$\subset int^*(cl^*(int^*(f^{-1}(E))))$$

hence

$f^{-1}(E) \subset int^*(cl^*(int^*(f^{-1}(E))))$. This shows that $f^{-1}(E)$ is $S.\alpha^* - I -$ open.

Since

$$f^{-1}(V) = X - f^{-1}(Y - V)$$
$$= X - f^{-1}(E)$$

This implies that $f^{-1}(V)$ is $S.\alpha^* - I -$ closed.

(3) ⇒ (1) if V be $S.\alpha^* - I -$ closed in Y, then $Y - V$ is $S.\alpha^* - I -$ open in Y. By (3) $f^{-1}(V)$ is $S.\alpha^* - I -$ closed in X and

$f^{-1}(V) \supset cl^*(int^*(cl^*(f^{-1}(V))))$

$X - f^{-1}(V) \subset X - cl^*(int^*(cl^*(f^{-1}(V))))$

$f^{-1}(Y - V) \subset int^*(cl^*(int^*(f^{-1}(Y - V))))$, so $f^{-1}(Y - V)$ is $S.\alpha^* - I -$ open in X.

Hence f is $S.\alpha^* - I -$ irresolute.

Theorem 2.17. If $f: (X, \tau, I) \to (Y, \sigma)$ is a $S.\alpha^* - I -$ continuous mapping and $g: (Y, \sigma) \to (Z, \mu)$ is a continuous mapping, then $g \circ f: (Y, \sigma, J) \to (Z, \mu)$ is a $S.\alpha^* - I -$ continuous mapping.

Proof. Let $V \in \mu$. Since g is continuous, then $g^{-1}(V) \in \sigma$. And since f is $S.\alpha^* - I -$ continuous, then $(g \circ f)^{-1}(V) = f^{-1}(g^{-1}(V))$ is $S.\alpha^* - I -$ open.

Hence $g \circ f$ is $S.\alpha^* - I -$ continuous.

Theorem 2.21. Let $f: (X, \tau, I) \to (Y, \sigma, J)$ and $g: (Y, \sigma, J) \to (Z, \mu,)$ be mappings. Then Then $g \circ f: (X, \tau, I) \to (Z, \mu)$ is $S.\alpha^* - I -$ continuous if g is $S.\alpha^* - I -$ continuous and f is $S.\alpha^* - I -$ irresolute.

Proof. Let $V \in \mu$. Since g is $S.\alpha^* - I -$ continuous, then $g^{-1}(V)$ is $S.\alpha^* - I -$ open. And since f is $S.\alpha^* - I -$ irresolute, then $(g \circ f)^{-1}(V) = f^{-1}(g^{-1}(V))$ is $S.\alpha^* - I -$ open. Hence $g \circ f$ is $S.\alpha^* - I -$ continuous.

Theorem 2.22. Let $f: (X, \tau, I) \to (Y, \sigma, J)$ and $g: (Y, \sigma, J) \to (Z, \mu, \delta)$ be mappings. Then $g \circ f: (X, \tau, I) \to (Z, \mu, \delta)$ is $S.\alpha^* - I -$ irresolute if both g and f are $S.\alpha^* - I -$ irresolute.

Proof. Let V is $S.\alpha^* - I -$ open in Z. Since g is $S.\alpha^* - I -$ irresolute, then $g^{-1}(V)$ is $S.\alpha^* - I -$ open. And since f is $S.\alpha^* - I -$ irresolute, then $(g \circ f)^{-1}(V) = f^{-1}(g^{-1}(V))$ is $S.\alpha^* - I -$ open. Hence $g \circ f$ is $S.\alpha^* - I -$ irresolute.

Theorem 2.23. Let $f: (X, \tau, I) \to (Y, \sigma)$ be a mapping. Then f is a $S.\alpha^* - I -$ continuous mapping if f is both $S.P^* - I -$ continuous and $S.S^* - I -$ continuous mappings.

Proof. It follows from (Theorem 10, [2]).

3- Strong α^*- I - Open (Closed) Mappings

Definition 3.1. A mapping $f: (X, \tau) \to (Y, \sigma, I)$ is called $S.\alpha^* - I -$ open if the $f(A)$ is a $S.\alpha^* - I -$ open set *in* (Y, σ, I), $\forall A \in \tau$.

Definition 3.2. A mapping $f: (X, \tau) \to (Y, \sigma, I)$ is called $S.\alpha^* - I -$ closed if $f(F)$ is a $S.\alpha^* - I -$ closed set *in* (Y, σ, I), for each closed set F in (X, τ).

As an example of $S.\alpha^* - I -$ open(closed) mappings, we give the following examples.

Example 3.3. Let $X = Y = \{a, b, c, d\}$, $\tau = \{\phi, X, \{d\}, \{a, c, d\}\}$, $\sigma = \{\phi, Y, \{c\}, \{b, c\}, \{b, c, d\}\}$, $I = \{\phi, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}, \{a, b. c\}\}$. If $f: (X, \tau) \to (Y, \sigma, I)$ defined by: $f(a) = d$, $f(b) = c$, $f(c) = a$, $f(d) = b$, then $f(X) = Y$, $f(\phi) = \phi$ and $f(A)$ is $S.\alpha^* - I -$ open $\forall A \in \tau$. Hence f is $S.\alpha^* - I -$ open.

Example 3.4. Let $X = Y = \{a, b, c, d\}$, $\tau = \{\phi, X, \{d\}, \{a, c, d\}\}$, $\sigma = \{\phi, Y, \{c\}, \{b, c\}, \{c, d\}, \{b, c, d\}\}$, $I = \{\phi, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}, \{a, b, c\}\}$ and $f: (X, \tau) \to (Y, \sigma, J)$ defined by: $f(a) = b$, $f(b) = a$, $f(c) = d$, $f(d) = c$. Then $f(X) = Y$, $f(\phi) = \phi$ and $f(F)$ is $S.\alpha^* - I -$ closed for each closed set $F \subset X$. Hence f is $S.\alpha^* - I -$ closed.

Theorem 3.5. $f: (X, \tau) \to (Y, \sigma, J)$ is $S.\alpha^* - I -$ open if and only if $\forall x \in X$ and each neighborhood U of x, there exists $S.\alpha^* - I -$ open set $V \subset Y$, $f(x) \in V$ such that $V \subset f(U)$.

Proof. Suppose that f is $S.\alpha^* - I -$ open. Then $\forall x \in X$ and each neighborhood U of x, there exists $U_0 \in \tau$ such that $x \in U_0 \subset U$. Since f is $S.\alpha^* - I -$ open, $V = f(U_0)$ is $S.\alpha^* - I -$ open in $Y$and $f(x) \in V \subset f(U)$. Conversely, let $U \in \tau$. Then $\forall x \in U$, there exists V_x is $S.\alpha^* - I -$ open in Y such that $f(x) \in V_x \subset f(U)$. Therefore, we obtain $f(U) = \cup \{V_x : x \in U\}$ and hence by (Theorem 17 (1), [2]), $f(U)$ is $S.\alpha^*$- I -open. This shows that f is $S.\alpha^*$- I -open.

Theorem 3.6 presents the relationship between $S.\alpha^* - I -$ open mapping, $S.\alpha^* - I -$ closure operators and $S.\alpha^* - I -$

interior operators.

Theorem 3.6 Let $f:(X,\tau)\to(Y,\sigma,J)$ be a mapping. Then the following statements are equivalent:

(1) f is $S.\alpha^*-I-$open.

(2) $f(int(A))\subset S.\alpha^*Iint(f(A)), \forall A\subset X$.

(3) $int(f^{-1}(V))\subset f^{-1}(S.\alpha^*Iint(V))$, $\forall\, V\subset Y$.

(4) $f^{-1}(S.\alpha^*Icl(V))\subset cl(f^{-1}(V)), \forall V\subset Y$.

(5) $f(int(A))\subset int^*(cl^*(int^*(f(A))))$, $\forall A\subset X$.

Proof. (1) ⇒ (2) Let f be $S.\alpha^*-I-$open and $A\subset X$, then $f(int(A))\subset f(A)$. This implies that

$$S.\alpha^*Iint(f(int(A)))\subset S.\alpha^*Iint(f(A))$$

, but $f(int(A))$ is $S.\alpha^*-I-$open. Therefore,

$$f(int(A))=S.\alpha^*Iint(f(int(A)))\subset S.\alpha^*Iint(f(A)).$$

(2) ⇒ (3) Let $V\subset Y$, then $f^{-1}(V)\subset X$ and by (2)

$$f(int(f^{-1}(V)))\subset S.\alpha^*Iint(f(f^{-1}(V)))\subset S.\alpha^*Iint(V).$$

So,

$$int(f^{-1}(V))\subset f^{-1}(f(int(f^{-1}(V))))\subset f^{-1}(S.\alpha^*Iint(V)).$$

(3) ⇒ (4) Let $V\subset Y$, then $Y-V\subset Y$ and by (3) we have

$$int(f^{-1}(Y-V))\subset f^{-1}(S.\alpha^*Iint(Y-V)).$$

Thus,

$$X-cl(f^{-1}(V))\subset f^{-1}(Y-S.\alpha^*Icl(V))=X-f^{-1}(S.\alpha^*Icl(V)).$$

Hence $f^{-1}(S.\alpha^*Icl(V))\subset cl(f^{-1}(V))$.

(4) ⇒ (5) Let $A\subset X$, then $Y-f(A)\subset Y$ and by (4), we get

$$f^{-1}(S.\alpha^*Icl(Y-f(A)))\subset cl(f^{-1}(Y-f(A)))$$

$$f^{-1}(Y-S.\alpha^*Iint(f(A)))\subset cl(X-f^{-1}(f(A)))$$

$$X-f^{-1}(S.\alpha^*Iint(f(A)))\subset X-int(f^{-1}(f(A))).$$

This implies that

$$int(f^{-1}(f(A)))\subset f^{-1}(S.\alpha^*Iint(f(A))).$$

Wherefore,

$$int(A)\subset int(f^{-1}(f(A)))\subset f^{-1}(S.\alpha^*Iint(f(A))).$$

Hence

$$f(int(A))\subset f(f^{-1}(S.\alpha^*Iint(f(A))))\subset S.\alpha^*Iint(f(A)).$$

Which shows that

$$f(int(A))\subset int^*(cl^*(int^*(S.\alpha^*Iint(f(A)))))\subset int^*(cl^*(int^*(f(A)))).$$

(5) ⇒ (1) Let $A\in\tau$, then by hypothesis we have

$$f(A)=f(int(A))\subset int^*(cl^*(int^*(f(A)))).$$

Hence $f(A)$ is $S.\alpha^*-I-$open. Thus f is $S.\alpha^*-I-$open.

Theorem 3.7. Let $f:(X,\tau)\to(Y,\sigma,J)$ be a bijective mapping and $\forall\, V\subset Y$. Then the following statements are equivalent:

(1) f is $S.\alpha^*-I-$closed.

(2) $f^{-1}(S.\alpha^*Icl(V))\subset cl(f^{-1}(V))$,

(3) $int(f^{-1}(V))\subset f^{-1}(S.\alpha^*Iint(V))$,

(4) $int(f^{-1}(V))\subset f^{-1}(int^*(cl^*(int^*(V))))$.

Proof. (1) ⇒ (2) Since f is $S.\alpha^*-I-$closed and $V\subset Y$, then

$$S.\alpha^*Icl(V) = S.\alpha^*Icl(f(f^{-1}(V)))$$
$$\subset S.\alpha^*Icl(f(cl(f^{-1}(V))))$$
$$\subset f(cl(f^{-1}(V))).$$

Therefore,

$$f^{-1}(S.\alpha^*Icl(V)) \subset f^{-1}(f(cl(f^{-1}(V))))$$
$$= cl(f^{-1}(V)).$$

(2) ⇒ (3) Let $V \subset Y$, then $Y - V \subset Y$ and by (2) we have

$f^{-1}(S.\alpha^*Icl(Y - V)) \subset cl(f^{-1}(Y - V))$.

So,

$X - f^{-1}(S.\alpha^*Iint(V)) \subset X - int(f^{-1}(V))$

Thus

$int(f^{-1}(V)) \subset f^{-1}(S.\alpha^*Iint(V))$.

(3) ⇒ (4) Let $V \subset Y$, then by (3)

$$int(f^{-1}(V)) \subset f^{-1}(S.\alpha^*Iint(V))$$
$$\subset f^{-1}(int^*(cl^*(int^*(S.\alpha^*Iint(V)))))$$
$$\subset f^{-1}(int^*(cl^*(int^*(V)))).$$

(4) ⇒ (1) Let F is closed in X, then $Y - f(F) \subset Y$ and by (4) we get

$$int(f^{-1}(Y - f(F)))$$
$$\subset f^{-1}(int^*(cl^*(int^*(Y - f(F)))))$$
$$X - cl(f^{-1}(f(F)))$$
$$\subset X - f^{-1}(cl^*(int^*(cl^*(f(F)))))$$

This implies that

$$f^{-1}(cl^*(int^*(cl^*(f(F)))))$$
$$\subset cl(f^{-1}(f(F))).$$ Therefore,

$$f^{-1}(cl^*(int^*(cl^*(f(F)))))$$
$$\subset cl(f^{-1}(f(F)))$$
$$= cl(F) = F.$$

Thus,

$$cl^*(int^*(cl^*(f(F))))$$
$$= f(f^{-1}(cl^*(int^*(cl^*(f(F))))))$$
$$\subset f(F).$$

This shows that $f(F)$ is $S.\alpha^* - I -$ closed. So, f is $S.\alpha^* - I -$ closed.

Theorem 3.8. If $f: (X, \tau) \to (Y, \sigma, I)$ is a $S.\alpha^* - I -$ open mapping, then $\forall\, V \subset Y$ and each closed set $F \subset X$ such that $f^{-1}(V) \subset F$, there exists a $S.\alpha^* - I -$ closed set $W \subset Y$ such that $V \subset W$ and $f^{-1}(W) \subset F$.

Proof. suppose that f is $S.\alpha^* - I -$ open. Let $V \subset Y$ and each closed set $F \subset X$ such that $F \supset f^{-1}(V)$. Then $X - F \in \tau$. And since f is $S.\alpha^* - I -$ open, then $f(X - F)$ is $S.\alpha^* - I -$ open. Hence $W = Y - f(X - F)$ is $S.\alpha^* - I -$ closed in Y. It follows from $f^{-1}(V) \subset F$ that $V \subset W$. Moreover, we obtain

$$f^{-1}(W) = X - f^{-1}(f(X - F))$$
$$\subset X - (X - F) = F.$$

Theorem 3.9. Let $f: (X, \tau) \to (Y, \sigma, I)$ be a mapping and $\forall\, A \subset X$. Then the following statements are equivalent:

(1) f is $S.\alpha^* - I -$ closed,

(2) $S.\alpha^*Icl(f(A)) \subset f(cl(A))$,

(3) $cl^*(int^*(cl^*(f(A)))) \subset f(cl(A))$.

Proof. (1) ⇒ (2) Let f be $S.\alpha^* - I -$ closed and $A \subset X$. Since $f(A) \subset cl(f(A))$, Then

$$S.\alpha^*Icl(f(A)) \subset S.\alpha^*Icl(f(cl(A)))$$
$$= f(cl(A))$$

(2) ⇒ (3) Let $A \subset X$, then by (2) we have

$$cl^*(int^*(cl^*(f(A))))$$
$$\subset cl^*(int^*(cl^*(S.\alpha^*Icl(f(A)))))$$
$$\subset S.\alpha^*Icl(f(A))$$
$$\subset f(cl(A)).$$

(3) ⇒ (1) Let A is closed in X, then by (3) we have

$cl^*(int^*(cl^*(f(A)))) \subset f(cl(A)) = f(A)$.

Hence $f(A)$ is $S.\alpha^* - I -$ closed. This shows that f is $S.\alpha^* - I -$ closed.

Theorem 3.10. A mapping $f:(X,\tau) \to (Y,\sigma,I)$ is $S.\alpha^* - I -$ open if and only if $f^{-1}(cl^*(int^*(cl^*(V)))) \subset cl(f^{-1}(V))$, $\forall\, V \subset Y$.

Proof. Let f be $S.\alpha^* - I -$ open. Since $cl(f^{-1}(V))$ is closed set containing $f^{-1}(V)$, $\forall V \subset Y$, then it follows from Theorem 3.8 that there exists $S.\alpha^* - I -$ closed $W \subset Y$ such that $V \subset W$ and $f^{-1}(W) \subset cl(f^{-1}(V))$. Since $V \subset W$, then

$$f^{-1}(cl^*(int^*(cl^*(V)))) \subset f^{-1}(cl^*(int^*(cl^*(W))))$$

and W is $S.\alpha^* - I -$ closed. Therefore,

$$f^{-1}(cl^*(int^*(cl^*(V)))) \subset f^{-1}(cl^*(int^*(cl^*(W)))) \subset f^{-1}(W) \subset cl(f^{-1}(V)).$$

Conversely, let $A \in \tau$, $Y - f(A) \subset Y$. Then by hypothesis

$$f^{-1}(cl^*(int^*(cl^*(Y - f(A))))) \subset cl(f^{-1}(Y - f(A)))$$

, this implies that

$$f^{-1}(Y - int^*(cl^*(int^*(f(A))))) \subset cl(f^{-1}(Y - f(A)))$$

$$X - f^{-1}(int^*(cl^*(int^*(f(A))))) \subset X - int(f^{-1}(f(A))).$$

Thus,

$int(f^{-1}(f(A))) \subset f^{-1}(int^*(cl^*(int^*(f(A)))))$. Wherefore,

$$A = int(A) \subset int(f^{-1}(f(A))) \subset f^{-1}(int^*(cl^*(int^*(f(A))))).$$

So,

$$f(A) \subset f(f^{-1}(int^*(cl^*(int^*(f(A)))))) \subset int^*(cl^*(int^*(f(A)))).$$

This shows that $f(A)$ is $S.\alpha^* - I -$ open. Hence f is $S.\alpha^* - I -$ open.

Theorem 3.11. A mapping $f:(X,\tau) \to (Y,\sigma,I)$ is $S.\alpha^* - I -$ open if

$$f(int^*(cl^*(int^*(A)))) \subset int^*(cl^*(int^*(f(A)))),\ \forall A \in \tau.$$

Proof. Let $A \in \tau$, then $A = int(A) \subset int^*(cl^*(int^*(A)))$.

And by hypothesis we get

$$f(A) \subset f(int^*(cl^*(int^*(A)))) \subset int^*(cl^*(int^*(f(A)))).$$

Hence $f(A)$ is $S.\alpha^* - I -$ open. This implies that f is $S.\alpha^* - I -$ open.

Theorem 3.12. If $f:(X,\tau) \to (Y,\sigma,I)$ is a $S.\alpha^* - I -$ closed mapping, then $\forall\, V \subset Y$ and $\forall\, A \in \tau$ such $f^{-1}(V) \subset A$, there exists $S.\alpha^* - I -$ open $W \subset Y$ and $V \subset W$ such that $f^{-1}(W) \subset A$.

Proof. let $W = Y - f(X - A)$. Since $f^{-1}(V) \subset A$, then $f(X - A) \subset Y - V$. And since f is $S.\alpha^* - I -$ closed, then W is $S.\alpha^* - I -$open and

$$f^{-1}(W) = X - f^{-1}(f(X - A)) \subset X - (X - A) = A.$$

Theorem 3.13. A mapping $f:(X,\tau) \to (Y,\sigma,I)$ is $S.\alpha^* - I -$ closed if

$$cl^*(int^*(cl^*(f(A)))) \subset f(cl^*(int^*(cl^*(A)))),\ \forall (X - A) \in \tau.$$

Proof. Let A is closed in X, then

$cl^*(int^*(cl^*(A))) \subset cl(A) = A.$

This implies that

$$f(cl^*(int^*(cl^*(A)))) \subset f(A)$$

and by hypothesis we obtain

$cl^*(int^*(cl^*(f(A))))$
$\subset f(\,cl^*(int^*(cl^*(A)))) \subset f(A)$.

Hence $f(A)$ is $S.\alpha^* - I -$ closed.

This shows that f is $S.\alpha^* - I -$ closed.

4- Strong $\alpha^* - I -$ Separation Axioms in Ideal Topological Spaces

Definition 4.1. A (X, τ, I) is called strong $\alpha^* - I - T_0$ space (briefly $S.\alpha^* - I - T_0$ space) if for each pair of distinct points x, y of X, there exists a $S.\alpha^* - I -$ open set containing one point but not the other.

Theorem 4.5. A (X, τ, I) is a $S.\alpha^* - I - T_0$ space if every singleton $\{x\}$ is a $S.\alpha^* - I -$ closed set, $\forall x \in X$.

Proof. Let $x \neq y$ such that $x, y \in X$ and $\{x\}$ is $S.\alpha^* - I -$ closed, then $X - \{x\}$ is $S.\alpha^* - I -$ open such that $y \in X - \{x\}$ and $x \notin X - \{x\}$. Hence (X, τ, I) is a $S.\alpha^* - I - T_0$ space.

Theorem 4.2. If (Y, σ, I) *is a* $S.\alpha^* - I - T_0$ space, then (X, τ, I) is a $S.\alpha^* - I - T_0$ space, when $f: (X, \tau, I) \rightarrow (Y, \sigma, J)$ be a $S.\alpha^* - I -$ irresolute injective mapping

Proof. Let (Y, σ, I) is a $S.\alpha^* - I - T_0$ space and $x, y \in X$ such that $x \neq y$ and $f(x) \neq f(y)$. Then there exists $S.\alpha^* - I -$ open $G \subset Y$ such that $f(x) \in G$ but $f(y) \notin G$. Since f is $S.\alpha^* - I -$ irresolute, then $f^{-1}(G)$ is $S.\alpha^* - I -$ open in (X, τ, I) such that $x \in f^{-1}(G)$ but $y \notin f^{-1}(G)$. Hence (X, τ, I) is $S.\alpha^* - I - T_0$ space.

Definition 4.3. A (X, τ, I) is called strong $\alpha^* - I - T_1$ space (briefly $S.\alpha^* - I - T_1$ space) if for each pair of distinct points x, y of X, there exists a pair of $S.\alpha^* - I -$ open sets one containing x but not y and the other containing y but not x.

Theorem 4.4. Let $f: (X, \tau, I) \rightarrow (Y, \sigma, J)$ be a $S.\alpha^* - I -$ irresolute injective mapping. Then (X, τ, I) is a $S.\alpha^* I - T_1$ space if (Y, σ, J) is a $S.\alpha^* I - T_1$ space.

Proof. Let (Y, σ, J) be a $S.\alpha^* - I - T_1$ space and $x, y \in X$ such that $x \neq y$ and $f(x) \neq f(y)$. Then there exists a pair of $S.\alpha^* - I -$ open sets $G, H \subset Y$ such that $f(x) \in G$, $f(y) \in H$, $f(x) \notin H$ and $f(y) \notin G$. Since f is $S.\alpha^* - I -$ irresolute, then $f^{-1}(G)$ and $f^{-1}(H)$ are $S.\alpha^* - I -$ open such that $x \in f^{-1}(G)$, $y \in f^{-1}(H)$, $x \notin f^{-1}(H)$ and $y \notin f^{-1}(G)$. Hence (X, τ, I) is a $S.\alpha^{*-} I - T_1$ space.

Theorem 4.5. A (X, τ, I) is a $S.\alpha^* - I - T_1$ space If every singleton $\{x\}$ is a $S.\alpha^* - I -$ closed set, $\forall x \in X$.

Proof. Let $x \neq y$ such that $x, y \in X$, $\{x\}$ is $S.\alpha^* - I -$ closed and $\{y\}$ is $S.\alpha^* - I -$ closed, then $X - \{x\}$ is $S.\alpha^* - I -$ open and $X - \{y\}$ is $S.\alpha^* - I -$ open such that $y \in X - \{x\}$ but $x \notin X - \{x\}$ and $x \in X - \{y\}$ but $y \notin X - \{y\}$. Hence X is a $S.\alpha^* - I - T_1$ space.

Theorem 4.6. Every $S.\alpha^* - I - T_1$space is a $S.\alpha^* - I - T_0$space.

Proof. Let (X, τ, I) be a $S.\alpha^* - I - T_1$ space and $x, y \in X$, $x \neq y$, then there exists a pair of $S.\alpha^* - I -$ open sets G, H such that $x \in G$ and $y \in H$ but $x \notin H$ and $y \notin G$. Since G is $S.\alpha^* - I -$ open such that $x \in G$ but $y \notin G$. Then (X, τ, I) is $S.\alpha^* - I - T_0$ space.

Definition 4.7. A space (X,τ,I) is called $S.\alpha^*-I-T_2$ space ($S.\alpha^*-I-$ hausdorff space) if for each pair of distinct points x,y of X there exists a pair of $S.\alpha^*-I-$ open sets G,H containing x and y respectively such that $G\cap H=\phi$.

Theorem 4.9. Let $f:(X,\tau,I)\to(Y,\sigma,J)$ be a $S.\alpha^*-I-$ irresolute injective mapping. Then (X,τ,I) is a $S.\alpha^*I-T_2$ space if (Y,σ,J) is a $S.\alpha^*I-T_2$ space.

Proof. Let (Y,σ,J) be a $S.\alpha^*-I-T_2$ space and $x,y\in X$ such that $x\neq y$ and $f(x)\neq f(y)$. Then there exists a pair of $S.\alpha^*-I-$ open sets $G,H\subset Y$ such that $f(x)\in G$, $f(y)\in H$ and $G\cap H=\phi$. Since f is $S.\alpha^*-I-$ irresolute, then $f^{-1}(G)$ and $f^{-1}(H)$ are $S.\alpha^*-I-$ open sets such that $x\in f^{-1}(G), y\in f^{-1}(H)$ and $f^{-1}(G)\cap f^{-1}(H)=\emptyset$. This shows that (X,τ,I) is a $S.\alpha^*-I-T_2$ space.

Theorem 4.10. If (X,τ,I) is a $S.\alpha^*I-T_2$ space, then for $x\neq y\in X$ there exists a $S.\alpha^*-I-$ open set G such that $x\in G$ and $y\notin S.\alpha^*Icl(G)$.

Proof. Let (X,τ,I) be a $S.\alpha^*I-T_2$ space. Let $x,y\in X$, then there exists a pair of $S.\alpha^*-I-$ open sets $G,H\subset X$ such that $x\in G, y\in H, G\cap H=\phi$. Therefore, $X-H$ is $S.\alpha^*-I-$ closed such that $S.\alpha^*Icl(G)\subset X-H$. Since $y\in H$, then $y\notin X-H$. Hence $y\notin S.\alpha^*Icl(G)$.

Theorem 4.11. Every $S.\alpha^*-I-T_2$space is a $S.\alpha^*-I-T_1$space.

Proof. Let (X,τ,I) be a $S.\alpha^*-I-T_2$ space and $x,y\in X$, $x\neq y$, then there exists a pair of $S.\alpha^*-I-$ open sets G,H such that $G\cap H=\phi$, $x\in G$ and $y\in H$ but $x\notin H$ and $y\notin G$. Since G is $S.\alpha^*-I-$ open such that $x\in G, y\notin G$ and H is $S.\alpha^*-I-$ open such that $y\in H, x\notin H$. Then (X,τ,I) is $S.\alpha^*-I-T_1$ space.

Definition 4.12. A (X,τ,I) is called strong α^*-I- regular space (briefly $S.\alpha^*-I-$ regular space) if $\forall\, x\in X$ and each $S.\alpha^*-I-$ closed set F is not containing x, there exists disjoint $S.\alpha^*-I-$ open sets G and H such that $x\in H$ and $x\notin G$, $F\subset H$.

Definition 4.13. A $S.\alpha^*-I-T_1$ regular space is called $S.\alpha^*-I-T_3$ space.

Definition 4.14. A space (X,τ,I) is called strong α^*-I- normal space (briefly $S.\alpha^*-I-$ normal space) if for each two disjoint $S.\alpha^*-I-$ closed sets $F_1,F_2\subset X$ there exists disjoint $S.\alpha^*-I-$ open sets G_1,G_2 such that $F_1\subset G_1$, $F_2\subset G_2$ and $G_1\cap G_2=\phi$.

Definition 4.15. A $S.\alpha^*-I-T_1$ normal space is called $S.\alpha^*-I-T_4$ space.

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تحليل الاستمرارية في الفضاءات التوبولوجية المثالية

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الملخص: في هذا البحث، قدمنا ودرسنا مفهوم جديد من مفاهيم الاستمرارية وهو (Strong $\alpha^* - I -$ continuity) وكذلك مفهومين جديدين من الرواسم المفتوحة والمغلقة على الفضاءات التوبولوجية المثالية وهما: (Strong $\alpha^* - I -$ closed mapping , Strong $\alpha^* - I -$open mapping) ، درسنا بعض خصائص هذه المفاهيم، كما درسنا العلاقة بين كلًا من هذه المفاهيم وبعض المفاهيم المعرفة سابقًا في الفضاء التوبولوجي والفضاء التوبولوجي المثالي. ودرسنا بديهيات فصل جديدة وهي: (Strong $\alpha^* - I -$ Separation Axioms) وبعض خصائصها.

الكلمات المفتاحية: الفضاءات التوبولوجية المثالية، الدوال المتصلة من النوع (strong $\alpha^* - I -$)، الدوال المفتوحة والمغلقة من النوع (Strong $\alpha^* - I -$) ، مسلمات الفصل من النوع (Strong $\alpha^* - I -$).