JEF/Journal of Education Faculties
Volume 17, Issue (1), 2023

مجلة كليات التربية – جامعة عدن
المجلد 17 ، العدد (1)، 2023 م

Other Notions of $\Lambda -$ Sets and V – Sets In Topological Spaces

Radhwan Mohammed Aqeel
Dept. of Mathematics, Faculty of Science
Aden University, Yemen
e-mail raqeel1976 @yahoo.com

Samah Mohammed Al-qadhi
Dept. of Mathematics, Faculty of Education
Aden University, Yemen
e-mail salqady047@gmial.com

Abstract: In this paper we have introduced and investigated a new notion in topological spaces called α-Λ-sets and α- V-sets, which are defined by the notion of ∧- sets and V- sets. We investigated the properties of the α-Λ-sets and α- V-sets. Also the achievement of the topology defined by these families of sets is obtained.

Keywords: Topological spaces, α-Λ-sets , α- V-sets.

1. Introduction

In 1986, Maki [6] continued the work of Levine [5]and Dunhem[2] on generalized closed sets and closure operators by introducing the notion of a-generalized Λ-set in a topological space (X,τ) and by defining an associated closure operator, i.e.
the ∧-closure operator. In this direction we shall introduce the notion of α-Λ-set and α-V-set in a given topological space and thus obtain new topologies defined by these families of sets. We also consider some of the fundamental properties of these new topologies.

1- Preliminaries.

Definition 2.1 A subset A of topological space (X,τ) is called:

(1) [9] Regular open, if $A = int(cl(A)$.

(2) [5] Semi-open, if $A \subset cl(int(A))$.

(3) [7] Pre-open, if $A \subset int(cl(A))$.

(4) [8] α-open, if $A \subset int(cl(int(A)))$.

The complement of a regular open (resp, semi-open, pre-open and α-open) set is called a regular closed (resp, semi-closed, pre- closed and α-closed).

Definition 2.2 [6] Let A be a set of a topological space (X , τ), Then

$A^{\wedge} = \cap\{U : A \subseteq U \text{ and } U \text{ is open}\}$ and

$A^{\vee} = \cup\{F \mid F \subseteq A \text{ and } F \text{ is closed}\}$.

Moreover, A is said to be ∧-set (or meet set) if $A = A^{\wedge}$ and A is said to be ∨-set
(or join set) if $A = A^{\vee}$.

Lemma 2.3 [6] Let (X,τ) be a topological space, A and B be subsets of X. Then the following hold

(1) $A^{\vee} \subseteq A \subseteq A^{\wedge}$.

(2) $A \subseteq B$ implise $A^{\vee} \subseteq B^{\vee}$ and $A^{\wedge} \subseteq B^{\wedge}$.

(3) If $A \in \tau$, then $A = A^{\wedge}$.

(4) $A^{\wedge\wedge} = A^{\wedge}$.

(5) $A^{\vee\vee} = A^{\vee}$.

(6) $(\bigcup_{i\in I} A_i)^{\wedge} = \bigcup_{i\in I} A_i^{\wedge}$.

(7) $(A^c)^{\wedge} = (A^{\vee})^c$.

(8) $(\bigcap_{i\in I} A_i)^{\wedge} \subset \bigcap_{i\in I} A_i^{\wedge}$.

(9) If $A^c \in \tau$, then $A = A^{\vee}$.

(10) $(\bigcap_{i\in I} A_i)^{\vee} = \bigcap_{i\in I} A_i^{\vee}$.

(11) $(\bigcup_{i\in I} A_i)^{\vee} \supset \bigcup_{i\in I} A_i^{\vee}$.

(12) If A_i is a $\wedge$-set ($i \in I$),
then $\bigcup_{i\in I} A_i$ is a $\wedge$-set .

(13) If A_i is a V-set ($i \in I$), then $\bigcap_{i\in I} A_i$
is a V-set .

(14) A is a $\wedge$-set if and only if
A^c is a V-set.

Definition 2.4 Let A be a subset of a topological space (X,τ). Then is called:

(1) $\Lambda_s - set$(resp, V_s − A) [1] if it is the intersection (resp, union) of semi-open (resp, semi-closed) sets.

(2) $\Lambda_p - set$(resp, V_p − sei) [4] if it is the intersection(resp, union) of pre-open (resp, pre-closed) sets.

(3) $\Lambda_\alpha - set$(resp, V_α − sei) [3] if it is the intersection (resp, union) of α-open (resp, α-closed) sets.

(4) A is called $\Lambda_s - set$ (resp, $V_s - set$) [1] if $A = \Lambda_s - set$ (resp, A=$V_s - set$).

(5) A is called $\Lambda_p - set$ (resp, $V_p - set$) [4] if A=$\Lambda_p - set$ (resp, A = $V_p - set$).

(6) A is called $\Lambda_\alpha - set$ (resp, $V_\alpha - set$) [3] if A=$\Lambda_\alpha - set$ (resp, A =$V_\alpha - set$).

Definition 2.5. [10] A subset A of a topological space (X,τ) is called:

(1) $Pre -\wedge - set$ ($resp.$ $pre -\vee - set$), if
$A \supset A^{\vee\wedge}$ (resp, $A \subset A^{\wedge\vee}$).

(2) $Semi -\wedge - set$ ($resp. semi -\vee - set$),
if $A \supset A^{\wedge\vee}$ (resp, $A \subset A^{\vee\wedge}$).

(3) α- $\wedge$ -sets.

Definition 3.1. A subset A of a topological space (X,τ) is called α-$\wedge$-set, if $A \supset A^{\wedge\vee\wedge}$.
We denote that all α- $\wedge$ -sets by α- $\wedge$ (x).

Proposition 3.2. Let (X, τ) be topological space, then for any subset A of X, the followings hold:

i. Every $\wedge$ -set is an α- $\wedge$ -set .

ii. Every α- $\wedge$ -set is a semi-$\wedge$ - set. .

iii. Every α- $\wedge$ -set is a pre-$\wedge$-set.

Proof .

i. Let A is a $\wedge$ -set , then $A = A^{\wedge}$.
So $A \supset A^{\wedge\vee} \Rightarrow (A)^{\wedge} \supset (A^{\wedge\vee})^{\wedge} \Rightarrow A \supset A^{\wedge\vee\wedge}$.
Thus A is an α- $\wedge$ -set .

ii. Let A is an α- $\wedge$ -set, then
$A \supset A^{\wedge\vee\wedge} \supset A^{\wedge\vee}$. Hence A is a semi-$\wedge$-set.

iii. Let A is an α- $\wedge$ -set, then
$A \supset A^{\wedge\vee\wedge} \supset A^{\vee\wedge}$. Hence A is a pre-$\wedge$-set .

The following diagram holds for any subset A of topological space (X, τ).

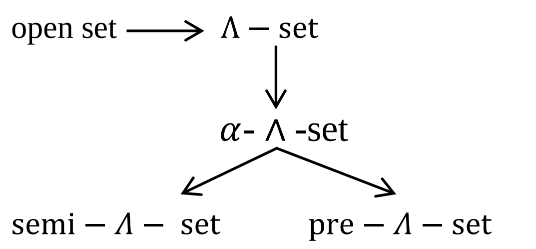

Diagram 1

Remark 3.3. The converse of this proposition 3.2. is not true as shown of the next examples.

Example 3.4 . Let X= { a, b , c , d} and

τ = {∅, {a}, {b},{a, b}, {a, c}, {a ,b, c}, X}

Then α- ∧ -sets = {∅, {a}, {b},{c},{a, b}, {a, c},{a, b ,c}, X} and ∧-sets= {∅, {a}, {b}, {a, b}, {a, c}, {a, b, c}, X }.

If we take A= {c}, then we get A is not ∧-set but it is an α- ∧ -set.

Example 3.5. Let X={a, b, c, d } and

τ = {∅, {a},{a, b}, {a, c, d}, X}. Then α-∧-sets = {∅ ,{a },{a ,b}, {a, c, d},X},

S- ∧-set ={∅ ,{a},{b},{a,b},{c,d},{a, c, d}, X} and P-∧-sets ={∅ ,{a},{c}, {a, b},{a ,c}, {a, d }, {a, b, c}, {a , b, d}, { a, c, d}, X }.

If we take A={ b } is a S- ∧-set but is not an α-∧-set . And if we take B= {c}, then we get B is not α-∧-set but it is a p- ∧ -set.

Theorem 3.6. Let (X, τ) be a topological space and A_i ∈ α- ∧ (x), then

$\cap$ **{** A_i : i ∈ I } ∈ α- ∧ (x), for each i ∈ I .

Proof. Let A_i is an α- ∧-set , then

$$A_i \supset {A_i}^{\wedge\vee\wedge} \quad \forall\, i \in I \text{, thus}$$

$$\cap_{i\in I} A_i \supset \cap_{i\in I} A_i^{\wedge\vee\wedge} \supset (\cap_{i\in I} A_i^{\wedge\vee})^{\wedge}$$

$$= (\cap_{i\in I} A_i^{\wedge})^{\vee\wedge} \supset (\cap_{i\in I} A_i)^{\wedge\vee\wedge}.$$

This shows that $\cap_{i\in I} A_i$ ∈ α- ∧ (X).

Lemma 3.7 . Let A be a subset of a space (X,τ). Then A is an α-∧-set in (X,τ) if and only if A is S-∧-set and P-∧-set in (X, τ).

Proof .

Let A ∈ α- ∧ (X) . By the definition of α-∧-set, we have A⊃$A^{\wedge\vee}$ and A⊃$A^{\vee\wedge}$. Therefore, we obtain A∈S-∧(X) ∩ P-∧(X) . Sufficiency , let A ∈ S- ∧ (X) ∩ P-∧ (X).

Since A ∈ P-∧ (X), A⊃ $A^{\vee\wedge}$ and hence it follows from A ∈ S-∧ (X) that

$$A \supset A^{\wedge\vee} \supset A^{\wedge\vee\vee\wedge} \supset A^{\wedge\vee\wedge}.$$

Therefore, we have A ∈ α- ∧ (X) .

Remark 3.8. The λ-open set and α- ∧-set are independent notions we can show that from the next example .

Example 3.9. Let X={a, b, c, d} and τ={∅, {a, b}, X}.

If we take A={c ,d} and B ={a}.

Then A is a λ-open but it is not α- ∧-set and B is not λ-open but it is an α-∧-set.

Remark 3.10 .The α- open set and α- ∧-set are independent notions we can show that from the next example:

Example 3.11. Let X ={ a ,b, c , d} and τ = {∅,{a},{b},{a ,b},{a,c},{a,b,c}, X}.

If A={c} and B ={a, b, d}. Then A is an α-∧-set but it is not α- open and B is not α-∧-set but it is an α- open set .

Remark 3.12. The $\wedge_\alpha$-set and α-∧-set are independent notions we can show that from the next example.

Example 3.13. Consider the topological space (X, τ) given in Example 3.4.

Hence, if A={c} and B ={a, b ,d}.Then A is an α-∧-set but it is not $\wedge_\alpha$-set and B is not α-∧ -set but it is $\wedge_\alpha$ - set .

Lemma 3.14. Every open set is an α-∧-set.

Proof. The Proof comes from the fact that, every open set is a ∧-set .

4- α-∨-sets

Definition 4.1. A subset A of topological spaces (X, τ) is called α-∨-set, if A⊂ $A^{\vee\wedge\vee}$.

We denote that all α- ∨-sets by α- ∨(X).

Proposition 4.2. Let (X,τ) be a topological space, the followings hold, for any subset A of X:

i. Every ∨-set is an α-∨-set.

ii. Every α-∨-set is a semi-∨-set .

iii. Every α-∨-set is a pre -∨-set.

Proof .

i. Let A is a ∨-set, then $A = A^{\vee} \Rightarrow$ A$\subset A^{\vee\wedge} \Rightarrow (A)^{\vee} \subset (A^{\vee\wedge})^{\vee} \Rightarrow A^{\vee} \subset A^{\vee\wedge\vee} \Rightarrow$ A $\subset A^{\vee\wedge\vee}$. Thus A is an α- ∨-set.

ii. Let A is an α-∨-set, then A$\subset A^{\vee\wedge\vee}$. Since A $\subset A^{\vee\wedge\vee} \subset A^{\vee\wedge} \Rightarrow$ A $\subset A^{\vee\wedge}$. Thus A is a s- ∨-set.

iii. Let A is an α- ∨-set, then A $\subset A^{\vee\wedge\vee} \Rightarrow A^{\vee} \subset$ A $\Rightarrow (A^{\vee})^{\wedge} \subset (A)^{\wedge} \Rightarrow A^{\vee\wedge} \subset A^{\wedge} \Rightarrow (A^{\vee\wedge})^{\vee} \subset (A^{\wedge})^{V} \Rightarrow A^{\vee\wedge\vee} \subset A^{\wedge\vee}$. Since A$\subset A^{\vee\wedge\vee} \Rightarrow$ A$\subset A^{\wedge\vee}$. Thus A is a p-∨-set.

The following diagram holds for any a subset A of topological space (X, τ)

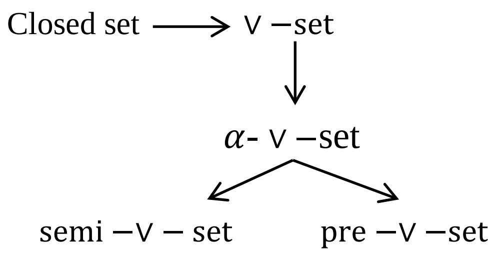

Diagram 2

Remark 4.3. The converse of these implications in Diagram 2 are not true in general as shown in the following examples**:**

Example 4.4. Let X = {a ,b, c, d},

τ ={∅,{a},{b}, {a, b}, {a, c}, {a ,b ,c}, X}. If we take A = {a, b, d }, then we get A is only if A is S -∨-set and p-∨-set in (X, τ) .

Proof . Let A∈ α- ∨(x) . By the definition of not an ∨-set but it is an α- ∨-sets.

Example 4.5. Let X = {a, b, c ,d },

τ = {∅ ,{a},{a, b},{a, c, d}, X }. If we take A= {a, b}, then we get A is not α- ∨-set but it is a semi- ∨-set. And if we take B={c}, then we get B is not an α- ∨-set but it is a pre- ∨-set .

Theorem 4.6. Let (X, τ) be a topological space and A⊂X, then the following statements are equivalent .

i. A is an α- ∨-set .

ii. A^{c} is an α- ∧-set .

Proof.

(i) ⟶ (ii) Let A be an α- ∨-set, then A $\subset A^{\vee\wedge\vee}$, implies that $(A)^{c} \supset (A^{\vee\wedge\vee})^{c} = (A^{c})^{\wedge\vee\wedge}$.

Hence A^{c} is an α- ∧-set .

(ii) ⟶ (i) Let A^{c} be an α- ∧-set, than $A^{c} \supset (A^{c})^{\wedge\vee\wedge}$, such that $(A^{c})^{c} \subset ((A^{c})^{\wedge\vee\wedge})^{c}$, implies that A $\subset \left((A^{\vee\wedge\vee})^{c} \right)^{C} = A^{\vee\wedge\vee}$.

Hence A is an α- ∨-set .

Theorem 4.7 . Let (X, τ) be a topological space and A_i be a subset of X. Then for each i∈ I , ∪ {A_i i∈ I }∈ α- ∨(X).

Proof. Let A_i be an α- ∨-set, then $A_i \subset A_i^{\vee\wedge\vee}$, thus $\cup_{i\in I} A_i \subset \cup_{i\in I} A_i^{\vee\wedge\vee} \subset (\cup_{i\in I} A_i^{\vee\wedge})^{\vee}$

$$= (\cup_{i\in I} A_i^{\vee})^{\wedge\vee}$$

$$\subset (\cup_{i\in I} A_i)^{\vee\wedge\vee} .$$

This shows that $\cup_{i\in I} A_i \in \alpha$- ∨(X).

Lemma 4.8. Let A be a subset of a space (X, τ). Then A is α-∨-set in (X, τ) if and α- ∨-set. We have A$\subset A^{\vee\wedge}$ and A$\subset A^{\wedge\vee}$. Therefore, we obtain A∈ s- ∨(x)∩ p- ∨(x).

Sufficiency. Let A ∈ s- ∨ (x) ∩ p- ∨(x). Since A ∈ p- ∨(x) , A ⊂ $A^{\wedge\vee}$ and hence it follows from A ∈ s- ∨(x) that A⊂ $A^{\vee\wedge}$ ⊂ $A^{\vee\wedge\wedge\vee}$ ⊂$A^{\vee\wedge\vee}$. Therefore we have A ∈ α- ∨(X) .

Remark 4.9. The λ-closed set and α- ∨-set are independent notions , we show that from the next example .

Example 4.10. Let X= { a, b, c, d}, τ ={∅,{a,b}, X}. If we take A={a ,b} and B={a, b, c}. Then A is an λ-closed set but is not α- ∨-set and B is an α- ∨-set but is not λ-closed.

Remark 4.11. The α- closed and α- ∨-set are impendent notions, we show that from the next example .

Example 4.12 . Consider the topological space (X, τ) given in Example 4.5. Hence, if we take A= {c} and B = {a, d} .Than A is an α- closed set but is not α- ∨-set and B is an α- ∨-set but is not α- closed set .

Remark 4.13. The $\vee_\alpha$ - set and α- ∨-set are independent notions, we show that from the next example .

Example 4.14. From Example4.10. Hence A is an $\vee_\alpha$ - set but is not α- ∨-set and B is an α- ∨-set but is not $\vee_\alpha$ - set .

Lemma 4.15 . Every closed set is a α-∨-set .

Proof. The proof comes from the fact that, every closed set is an ∨-set.

5. Some Operator via ∧ -sets and ∨-sets

Definition 5.1 A subset A of a topological space (X,τ) is called $\wedge^\alpha$-set if intersection of all α- ∧-sets containing A .

$\wedge^\alpha$ (A) = ∩ { G: G ⊇ A. G ∈ α- ∧(X) }.

Example 5.2. Let X={a, b, c, d}, τ = {∅ ,{b},{c},{b, c}, X}.

Then α-∧(X) = {∅ ,{b},{c}, {b, c}, X}.

$\wedge^\alpha$ ({a})=X , $\wedge^\alpha$ ({c}) = {c},

$\wedge^\alpha$({a, d}) = X , $\wedge^\alpha$ ({a, c, d}) = X and $\wedge^\alpha$ (X) = X.

Theorem 5.3. For subsets A, B and $A_i (i \in I)$ of a topological space (X, τ), the following hold

i. A ⊆ $\wedge^\alpha$ (A)
ii. If A ⊆ B . Then $\wedge^\alpha$(A) ⊆ $\wedge^\alpha$ (B) .
iii. $\wedge^\alpha$ ($\wedge^\alpha$(A))=$\wedge^\alpha$(A).
iv. If A ∈ α- ∧((x) , then A= $\wedge^\alpha$(A).
v. $\wedge^\alpha$(∪{A_i li ∈ I})=∪{$\wedge^\alpha$(A_i) I i ∈ I }.
vi. $\wedge^\alpha$(∩{A_i li ∈I})⊂∩ {$\wedge^\alpha${(A_i)li ∈I}.

Proof.

i. It is clear by Definition 5.1.

ii. Suppose that x∉ $\wedge^\alpha$(B). Then there exists a subset G ∈ α- ∧(X) such that B ⊂ G with x ∉ G such that x ∉ B, since A ⊂ B then x ∉ $\wedge^\alpha$(A) and thus $\wedge^\alpha$(A) ⊂ $\wedge^\alpha$ (B) .

iii. It follows from (i) and (ii) that $\wedge^\alpha$(A)⊆$\wedge^\alpha$($\wedge^\alpha$(A)). If x∈$\wedge^\alpha$(A), then there exists G ∈ α-∧(X) such that A∈ G and x ∉ G hence $\wedge^\alpha$(A) ⊂ G and so we have X ∉ $\wedge^\alpha$($\wedge^\alpha$(A)). Then ($\wedge^\alpha$($\wedge^\alpha$ (A)) =$\wedge^\alpha$(A) .

iv. Let A ∈ α- ∧(X). Since A is the least α- ∧- set containing itself, then $\wedge^\alpha$ (A) =A. Form (ii),

$\wedge^{\alpha}(A_i)\subset\wedge^{\alpha}(\cup\{A_i\ li\in I\})$ implies that $\cup\{\wedge^{\alpha}(A_i)\ li\in I\}\subset\wedge^{\alpha}(\cup\{A_i\ li\in I\})$.

Conversely suppose that exists a point x such that $x\notin\wedge^{\alpha}(\cup\{A_i\ li\in I\})$. Then there exists an α-$\wedge$-set G such that $\cup\{A_i\ li\in I\}\subset G$ and $x\notin G$. Thus for each $i\in I$ we have $x\notin\wedge^{\alpha}(A_i)$.

This implies that $x\notin(\cup\ \{\wedge^{\alpha}(A_i)\ li\in I\})$.

v. Suppose that there exists a point x such that $x\notin\cap\{\wedge^{\alpha}(A_i) li\in I\}$ then, there exists $i_0\in I$ such that $x\notin\wedge^{\alpha}(A_{i0})$ and there exists an α- $\wedge$-set G such that $x\notin G$ and $A_{i0}\subset G$. We have $\cap\{\wedge^{\alpha}(A_{i0}) li\in I\}\subset A_{i0}\subset G$ and $x\notin G$. Therefore, $x\notin\wedge^{\alpha}(\cap\{A_i\}\ li\in I\})$.

Remark 5.4 In general we have $\wedge^{\alpha}(A_1\cap A_2)\neq\wedge^{\alpha}(A_1)\cap\wedge^{\alpha}(A_2)$. This can be shown by the following example:

Example 5.5. Let X={a, b, c, d}, τ ={$\emptyset$, {a},{b},{a, b}, {a, c}, {a ,b, c},X }. Now put A_1 ={a} and A_2= {b}. Then $\wedge^{\alpha}(A_1\wedge A_2)=\emptyset\neq\{b\}=\wedge^{\alpha}(A_1)\cap\wedge^{\alpha}(A_2)$.

Theorem 5.6 A subset A of a topological space (X, τ) is called an α- $\wedge$-set if and only if A= $\wedge^{\alpha}$ (A).

Proof. The proof comes from Theorem 3.6. and Theorem 5.3.

Now we introduce the notions of revised weak forms of $\boldsymbol{\alpha}$- $\vee$-sets .

Definition 5.7. A subset of A of a topological space (X, τ) is called $\vee^{\alpha}$- set if union of all α- $\vee$-sets contained in A .

$\vee^{\alpha}(A)=\cup\{S\setminus S\subseteq A, S\in\alpha\text{-}\vee(X)\}$.

Example 5.8. Let X = {a, b, c, d}, τ={ $\emptyset$, X, {b},{c},{b, c}}. Hence $\vee^{\alpha}(\{a\})=\emptyset$, $\vee^{\alpha}(\{a, d\})=\emptyset$, $\vee^{\alpha}(\{a, c, d\})=\{a, c, d\}$ and $\vee^{\alpha}(X)=X$.

Theorem 5.9. For subsets A , B and $A_i(i\in I)$ of a topological space (X, τ), the following held

i. $\vee^{\alpha}(A)\subseteq A$.
ii. If $A\subseteq B$, then $\vee^{\alpha}(A)\subset(B)$.
iii. $\vee^{\alpha}(\vee^{\alpha}(A))=\vee^{\alpha}(A)$.
iv. $(\vee^{\alpha}(A^{\alpha}))^{c}=\wedge^{\alpha}(A^{C})$.
v. If $A\in\alpha$- $\vee$(X), then $A=\vee^{\alpha}(A)$.
vi. $\vee^{\alpha}(\cap\{A_i\ li\in I\})=\cap\{\vee^{\alpha}(A_i)\} li\in I\}$
vii. $\vee^{\alpha}(\cup(\{A_i\ li\in I\})\supset\cup\{\vee^{\alpha}(A_i)\ li\in I\}$.

Proof .

i. It's clear by Definition 5.7.

ii. Suppose that $x\notin\vee^{\alpha}(B)$. Then there exists a subset $S\in\alpha$- $\vee$(X) such as that $S\subset B$ with $x\notin B$ such that $x\notin S$ since $A\subset B$, then $x\notin\vee^{\alpha}(A)$ and thus $\vee^{\alpha}(A)\subseteq\vee^{\alpha}(B)$.

iii. It follows from (i) and (ii) that $\vee^{\alpha}(A)\subseteq\vee^{\alpha}(\vee^{\alpha}(A))$. If $x\notin\vee^{\alpha}(A)$, then there exists $S\in\alpha$- $\vee$(X) such that $A\supset S$. $x\notin S$ hence $\vee^{\alpha}(A)\subset S$ and so we have $x\notin\vee^{\alpha}(\vee^{\alpha}(A))$. Then $\vee^{\alpha}(\vee^{\alpha}(A))=\vee^{\alpha}(A)$.

iv. $(\vee^{\alpha}(A))^{c}=(\cup\{S: S\supseteq A, S\in\alpha\text{-}\vee(X)\})^{c}$
$=\cap\{S^{c}: S^{c}\subseteq A^{c}, S^{c}\in\alpha\text{-}\wedge(X)\}$

Put $S^{c}=G$, then we have

$(\vee^{\alpha}(A))^{c}=\cap\{G: G\subseteq A^{c}, G\in\alpha\text{-}\wedge(X)\}$
$=\wedge^{\alpha}(A^{c})$.

To prove (v), let B an α- $\vee$-set in (X, τ), then $B^{c}\in\alpha$- $\vee$ (X, τ). Thus $B^{c}=(\vee^{\alpha}(B))^{c}$.

Hence $B=\vee^{\alpha}(B)$.

To prove (vi) , by theorem 5.9. (v) that

$\wedge^{\alpha}(\cup\{ A_i \ li \in \mathrm{I}\}) = (\cup \{\wedge^{\alpha} (A_i) \ i \in \mathrm{I} \})$.

Take complement for both such that

$\wedge^{\alpha}(\cup\{ A_i \ li \in \mathrm{I}\})^c = (\cup\{ \wedge^{\alpha} (A_i) \ \mathrm{li} \in \mathrm{I} \})^c$.

Hence $\vee^{\alpha}(\cap\{ {A_i}^c li \in \mathrm{I}\}) = \cap\{\vee^{\alpha}(A_i)^c \mathrm{l}\ i \in \mathrm{I}\}$.

To prove (vii) , by using statement (iv) and (v) we have

$$\wedge^{\alpha}(\cup \{ A_i \ \mathrm{l}\ i \in \mathrm{I}\}) = (\wedge^{\alpha}(\cup\{ A_i \ li \in \mathrm{I}\})^c)^c$$
$$= (\wedge^{\alpha} (\cap\{ A_i^c \ \ li \in \mathrm{I}\}))^c$$
$$\supset (\cap\{\wedge^{\alpha} (A_i^c) \ li \in \mathrm{I}\})^c$$
$$= (\cap \{ \vee^{\alpha} (A_i)^c) \ li \in \mathrm{I}\})^c$$
$$= \cup\{ ((\vee^{\alpha} (A_i)^c)^c li \in \mathrm{I}\}$$
$$= \cup \{ \vee^{\alpha} \ (A_i) \ li \in \mathrm{I}\} .$$

Remark 5.10. In the Theorem 5.9. part (vii) , then in conclusion can not be replaced by equality , as the following example .

Example 5.11. From Example 5.8. if we put A ={a} and B={d}. Then $\vee^{\alpha}(A \cup B)$ ={a, b}, but $\vee^{\alpha}$ (A) $\cup$ $\vee^{\alpha}$ (B) = $\emptyset$.

Theorem 5.12. A subset A of a topological space (X , τ) is called an α- $\vee$-set if

A = $\vee^{\alpha}$ (A) .

Proof . The proof comes from Theorem4.7, and Theorem 5.9.

References

[1] Caldas, M. and Dontchev, J.(2000). G.Λ_s-sets and G.V_s-sets, Mem. Fac. Sci. Kochi Univ.(Math.) 21, 21– 30.

[2] Dunham, W. (1977). $T1\backslash$ 2 spaces, Kyungpook math. J. , 17, 161−169.

[3] El-Sharkasy, M. (2015). On Λ_{α}-sets and the associated topology $\tau^{\Lambda_{\alpha}}$, Journal of the Egyptian Mathematical Society, 23(2), 371-376.

[4] Ganster, M., Jafar, S. and Noiri, M. (2002). On pre- Λ -sets and pre-V-sets, Acta Math. Hungar 95 (4) 337–343.

[5] Levine, N. (1963). Semi-open sets and semi-continuity in topological spaces, Amer. Math. Monthly, 70, 36 -41.

[6] Maki, H.(1986). Generalized Λ-sets and the associated closure operator, The Special Issue in Commemoration of Prof. Kazusada IKEDA' Retirement 1(10),139- 146.

[7] Mashhour, A.S., Abd El-Monsef, M.E. and El-Deeb, S.N. (1982). On precontinuous and weak precontinuous mapping, Proc. Math. Phys. Soc. Egypt (53) 47–53.

[8] Njastad, O., (1965).On some classes of nearly open sets, Pacific J. Math., 15, 961-970.

[9] Stone, M. H., (1937). Applications of the theory boolean rings to the general topology, Trans. A.M.S. 41, 375 – 481.

[10] Yaslam, M.H., (2023). *On Some Generalization of* , Λ - Sets and $\vee$ - *Sets IN Topological* Spaces, M.Sc. thesis, Faculty of Education, Aden University, Yemen,

مفاهيم أخرى في الفضاءات الطوبولوجية للمجموعات

$\boldsymbol{Sets\text{-}\Lambda}$ و $\boldsymbol{Sets\text{-}V}$

رضوان محمد سالم عقيل
قسم الرياضيات، كلية العلوم
جامعة عدن، اليمن
e-mail salqady047@gmial.com

سماح محمد أحمد القاضي
قسم الرياضيات، كلية التربية
جامعة عدن، اليمن
e-mail **raqeel1976 @yahoo.com**

الملخص:
في هذا البحث ، قدمنا ودرسنا مفاهيم جديدة في الفضاءات الطوبولوجية تسمى α-Λ-sets و α-V-sets ، والتي تم تعريفها من خلال مفاهيم Λ-sets- و V-sets, لقد قمنا بدراسة العديد من خواصها .
كما تم دراسة علاقة هذه المفاهيم بالمفاهيم الطوبولوجية السابقة .

الكلمات المفتاحية:
الفضاءات الطوبولوجية، المجموعات α-V-sets و α-Λ-sets