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Study of Representation of the Abel-Poisson Summability Based on Fractional Part Function and its Generalization

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Abstract:

The Poisson Summability used Abel's method in order to studying converges uniformly Fourier series. In this paper, we give instead of a fixed **r** in Abel's method, take of fractional Part function by their Fourier series on [-L,L], we obtain the new results. Finally,we generalized Poisson summability and studying converges uniformly on [-L,L].

Keywords: Poisson Summability, Fourier Series, Fractional Part Function, Dirichlet Kernel, Poisson Kernel.

1. Introduction

Given $\sum_{n=1}^{\infty} a_n = A$ be convergent , hence $\sum_{n=1}^{\infty} r^n a_n = A(r)$ is also convergent for all $0 \le r < 1$ and $\lim_{r\to 1^-} A(r) = A$.

Is called Abel's method (see[8],[11],[6]). Poisson used Abel's method in order to recover the original function of Fourier Series, (see[5], [10], [11]) which gave as following:

Let $f \in [-\pi, \pi]$ and 2π-periodic function: Then

$$A_r f(x) = \frac{a_0}{2} + \sum_{k=1}^{\infty} r^k (a_k coskx + b_k sinkx) . \qquad (1.1)$$

The coefficients at the above are

$a_0 = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\, dx$,

$a_k = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\, cos(kx)\, dx$ and

$b_k = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\, sin(kx)\, dx$.

Poisson summation is very important applications in physics and Engineering.

Let $\mathcal{L}_{[-L,L]}^{2L} = \mathcal{L}$, define as space of every continuous and 2L- Periodic function.

Now, we introduce in this paper, instead of a fixed **r** take Fraction part, in order to we reach for positive results (see the third section for details) and give theorem, it is generalized the Poisson summation(see section 4 for details).

Now, we introduce some mathematical preliminaries that are very important.

1. Preliminaries

Definition 1.1. [2]. A series of the form

$$f(x)=\frac{a_0}{2}+\sum_{n=1}^{\infty}\left(a_n\cos\left(\frac{n\pi x}{L}\right)+b_n\sin\left(\frac{n\pi x}{L}\right)\right). \quad (2.1)$$

is called a trigonometric series.

Definition 1.2.([2],[12]).If the trigonometric series in (2.1) convergent uniform to the function f in [-L,L], under series of the form:

$$f(x)\sim\frac{a_0}{2}+\sum_{n=1}^{\infty}\left(a_n\cos\left(\frac{n\pi x}{L}\right)+b_n\sin\left(\frac{n\pi x}{L}\right)\right),$$

then $f(L)=f(-L)$ and the coefficients are

$$a_0=\frac{1}{L}\int_{-L}^{L}f(x)dx,$$

$$a_n=\frac{1}{L}\int_{-L}^{L}f(x)\cos\left(\frac{n\pi x}{L}\right)dx, \text{ and}$$

$$b_n=\frac{1}{L}\int_{-L}^{L}f(x)\sin\left(\frac{n\pi x}{L}\right)dx.$$

Definition 1.3. (Dirichlet's Test([5],[3])).

Let $\{f_k\}$ and $\{g_k\}$ is two function sequences**1.** in X such that.

i) There is m>0 and

ii) $\left|\sum_{k=1}^{n}f_k(x)\right|\le m, \forall m\ge 1$, $x\in X$,

iii) $g_{k+1}(x)\le g_k(x), \forall k, \forall x\in X$,

iv) $\{g_k\}\to 0$ uniformly on X. Then

$\sum_{k=1}^{\infty}f_k g_k$ is uniformly convergent in X.

Definition 1.4. ([13],[14],[15],[16])

The floor function of real number x is define by symbol $\lfloor x\rfloor$ that satisfies the following $\lfloor x\rfloor\le x<\lfloor x\rfloor+1$;

the fraction part of x denoted by symbol $\{x\}$ that satisfies x=$\lfloor x\rfloor+\{x\}$;

the ceiling function of x is denoted by symbol $\lceil x\rceil$ *that* fits $x\le\lceil x\rceil<x+1$.

Lemma 1.1 (Riemann-Lebesgue ([5],[1])).

Given f is an integrable function in $[a,b]$ and $\lambda\in R$. Then

$$\lim_{\lambda\to\infty}\int_a^b f(t)\sin(\lambda t)dt$$

$$=\lim_{\lambda\to\infty}\int_a^b f(t)\cos(\lambda t)dt=0 .$$

Theorem.1.1. ([1],[4],[7]).

Let m≥ 0, the Dirichlet's kernel defined by:

$$D_m(t)=\frac{1}{2}+\sum_{k=1}^{m}\cos\left(\frac{k\pi t}{L}\right)=\frac{1}{2}\sum_{k=-m}^{m}e^{i\left(\frac{k\pi t}{L}\right)} . \quad (2.1)$$

Proof: Since

$$\frac{1}{2}\sum_{k=-m}^{m}e^{i\left(\frac{k\pi t}{L}\right)}=\frac{1}{2}+\sum_{k=-1}^{-m}e^{i\left(\frac{k\pi t}{L}\right)}+\sum_{k=1}^{m}e^{i\left(\frac{k\pi t}{L}\right)}$$

$$=\frac{1}{2}+\sum_{k=1}^{m}e^{-i\left(\frac{k\pi t}{L}\right)}+\sum_{k=1}^{m}e^{i\left(\frac{k\pi t}{L}\right)}, \text{ thus}$$

$$\frac{1}{2}\sum_{k=-m}^{m}e^{i\left(\frac{k\pi t}{L}\right)}=\frac{1}{2}+\sum_{k=1}^{m}\cos\left(\frac{k\pi t}{L}\right).$$

2. Abel - Poisson Summability

Definition 2.1. Let $\sum_{n=1}^{\infty}a_n=A$ be a convergent numerical series, then:

- The function $A(\{y\})=\sum_{n=1}^{\infty}\{y\}^n a_n$ is well defined (convergent) $\forall, 0\le\{y\}<1$.
- $\lim_{\{y\}\to 1^-}A(\{y\})=A$, where $\{y\}$ be the Fractional part function from Definition (1.4), y $\in R$.

We introduce this Abel's method to Fourier series. Consist the following , where the coefficients in Definition (1.2):

$$A_{\{y\}}f(x)=\frac{a_0}{2}++\sum_{k=1}^{\infty}\{y\}^k\left(a_k\cos\left(\frac{k\pi x}{L}\right)+b_k\sin\left(\frac{k\pi x}{L}\right)\right). \quad (3.1)$$

Which we have to study its limit

$A_{-1}f(x) = \lim_{\{y\}\to 1^-} A_{\{y\}}f(x)$, what we call the Abel summation of $f(x)$.

Using the exponential expression form equation(**3.1**), we obtain the following:

$$A_{\{y\}}f(x) = \sum_{k=-\infty}^{\infty}\{y\}^{|k|}\,C_k e^{i\left(\frac{k\pi x}{L}\right)}. \quad (3.2)$$

Indeed,

$$\forall k \geq 0, c_{\pm} = \frac{1}{2}a_k \pm \frac{1}{2i}b_k \Rightarrow \frac{1}{2}(\{y\}^k a_k) \pm \frac{1}{2i}(\{y\}^k b_k) = \{y\}^k c_{\pm k}$$, thus

$$\{y\}^k\left(\frac{1}{2}a_k \pm \frac{1}{2i}b_k\right) = \{y\}^k c_{\pm k} .$$

We want to express it in an integral form. Let $f \in \mathcal{L}$, hence C_k be bounded by the Lemma (1.1). Thus the series in equation (2.2) is continuous for every $\lfloor y\rfloor \leq y < 1 + \lfloor y\rfloor$ and uniform convergence (see [l],[9]) in $\lfloor y\rfloor \leq y \leq 1 + \lfloor y\rfloor - \varepsilon$, for any $\varepsilon > 0$, with these observations, the computation bellow are justified.

$$\sum_{k=-\infty}^{\infty} C_k e^{i\left(\frac{k\pi x}{L}\right)}\{y\}^{|k|}$$
$$= \sum_{k=-\infty}^{\infty}\frac{1}{2L}\int_{-L}^{L} f(t)e^{-i\left(\frac{k\pi t}{L}\right)}\, e^{i\left(\frac{k\pi x}{L}\right)}\{y\}^{|k|}dt$$
$$= \frac{1}{2L}\int_{-L}^{L} f(t)\sum_{k=-\infty}^{\infty} e^{-i\left(\frac{k\pi t}{L}\right)}e^{i\left(\frac{k\pi x}{L}\right)}\{y\}^{|k|}dt$$
$$= \frac{1}{L}\int_{-L}^{L} f(t)\frac{1}{2}\sum_{k=-\infty}^{\infty} e^{i\frac{k\pi}{L}(x-t)}\{y\}^{|k|}dt.$$

Definition 2.2. We define the Poisson kernel as the following:

$$P_{\{y\}}(t) = \frac{1}{2}\sum_{k=-\infty}^{\infty} e^{i\left(\frac{k\pi t}{L}\right)}\{y\}^{|k|},$$

therefore,

$$A_{\{y\}}f(x) = \frac{1}{L}\int_{-L}^{L} f(t)p_{\{y\}}(x-t)dt .$$

Using some alternative relations for the Poisson kernel. In the first plane, by Euler's formula

$$P_{\{y\}}(t) = \frac{1}{2}\sum_{k=-\infty}^{\infty} e^{i\left(\frac{\pi k t}{L}\right)}\{y\}^{|k|}$$
$$= \frac{1}{2}\sum_{k=-\infty}^{\infty}\left(cos\left(\frac{\pi k t}{L}\right) + isin\left(\frac{\pi k t}{L}\right)\right)\{y\}^{|k|}$$
$$= \frac{1}{2}\sum_{k=-\infty}^{\infty}\{y\}^{|k|}cos\left(\frac{\pi k t}{L}\right) = \frac{1}{2} + \sum_{k=1}^{\infty}\{y\}^{|k|}cos\left(\frac{\pi k t}{L}\right),$$

where we have used sine and cosine parities, since summation of one to infinity, then, we exclude the absolute value. Thus

$$P_{\{y\}}(t) = \frac{1}{2} + \sum_{k=1}^{\infty}\{y\}^k cos\left(\frac{\pi k t}{L}\right). \quad (3.3)$$

We find the partial sums of the equation (3.3).

$$2P_{\{y\}}(t) = \sum_{k=-\infty}^{\infty} e^{i\left(\frac{k\pi t}{L}\right)}\{y\}^{|k|}$$
$$= \sum_{k=-\infty}^{0} e^{i\left(\frac{k\pi t}{L}\right)}\{y\}^{-k} + \sum_{k=0}^{\infty} e^{i\left(\frac{k\pi t}{L}\right)}\{y\}^{k} - 1$$
$$2P_{\{y\}}(t) = \sum_{k=0}^{\infty} e^{-i\left(\frac{k\pi t}{L}\right)}\{y\}^k + \sum_{k=0}^{\infty} e^{i\left(\frac{k\pi t}{L}\right)}\{y\}^k - 1$$
$$2P_{\{y\}}(t) = \frac{1}{1-\{y\}e^{-i\left(\frac{\pi t}{L}\right)}} + \frac{1}{1-\{y\}e^{i\left(\frac{\pi t}{L}\right)}} - 1$$
$$2P_{\{y\}}(t) = \frac{1-\{y\}^2}{1-2\{y\}cos\left(\frac{\pi t}{L}\right)+\{y\}^2},$$

if we multiply by $\frac{1}{2}$ at the above, we obtain the formula

$$P_{\{y\}}(t) = \frac{1-\{y\}^2}{2\left(1-2\{y\}cos\left(\frac{\pi t}{L}\right)+\{y\}^2\right)} . \quad (3.4)$$

Now, let us give some properties of the Poisson kernel.

Theorem 2.2.

1) $P_{\{y\}}$ is an even, nonnegative and 2L-periodic function.

2) $\frac{1}{L}\int_{-L}^{L} P_{\{y\}}(t)dt$=1.

3) $\forall\delta > 0, P_{\{y\}}(t)$ tends uniformly to zero as $\{y\} \to 1^-$ in $[-L, L] - (-\delta, \delta)$.

Proof:

1) From expressions in (3.4) , we observe that it is nonnegative. form (3.1) we see that, it is clear that even and $2L$ −periodic.

2) From the equation (3.3),we take integral from L to –L , we obtain

$\frac{1}{L}\int_{-L}^{L} P_{\{y\}}(t)dt$

$=\frac{1}{L}\int_{-L}^{L}\left(\frac{1}{2}+\sum_{k=1}^{\infty}\{y\}^{k}cos\left(\frac{\pi kt}{L}\right)\right)dt$

$=\frac{1}{L}\int_{-L}^{L}\frac{1}{2}dt = 1.$

3) Given $\delta > 0$, we have that for every $\delta \leq |t| \leq L$:

We can bound the denominator by.

$$2\left(1-2\{y\}cos\left(\frac{\pi t}{L}\right)+\{y\}^{2}\right)$$
$$= 2\left(1-2\{y\}+\{y\}^{2}-2\{y\}cos\left(\frac{\pi t}{L}\right)+2\{y\}\right)$$
$$= 2\left[(1-\{y\})^{2}+2\{y\}\left(1-cos\left(\frac{\pi t}{L}\right)\right)\right]$$
$$\geq 2\left[(1-\{y\})^{2}+2\{y\}\left(1-cos\left(\frac{\pi \delta}{L}\right)\right)\right],$$

therefore

$$P_{\{y\}}(t) \leq \frac{1-\{y\}^{2}}{2\left[(1-\{y\})^{2}+2\{y\}\left(1-cos\frac{\pi\delta}{L}\right)\right]} \quad .$$

From above equation, it is tends to zero where $\{y\} \to 1^{-}$ independent of **t** , that is uniform. Thus

$$\frac{1-\{y\}^{2}}{2\left[(1-\{y\})^{2}+2\{y\}\left(1-cos\frac{\pi\delta}{L}\right)\right]} \xrightarrow{\{y\}\to 1^{-}} 0 \quad .$$

Now, we give alternative integral for the Abel's series of the Poisson kernel is:

$$A_{\{y\}}f(x) = \frac{1}{L}\int_{-L}^{L} f(x-t)p_{\{y\}}(t)dt. \quad (3.5)$$

The above integral can be written as.

$$A_{\{y\}}f(x) = \frac{1}{L}\int_{0}^{L} p_{\{y\}}(t)[f(x-t)+f(x+t)]dt. \quad (3.6)$$

Let us now give theorem about Poisson summability.

Theorem 2.3.

Given $f \in \mathcal{L}$ be an integrable function.

If f has got lateral limit at x, then.

$$\lim_{\{y\}\to 1^{-}} A_{\{y\}}f(x) = \frac{1}{2}[f(x^{+})+f(x^{-})].$$

Proof:

From integral (3.6), we get

$$A_{\{y\}}f(x)-\frac{1}{2}[f(x^{+})+f(x^{-})]$$
$$=\frac{1}{L}\int_{0}^{L} p_{\{y\}}(t)\,[f(x-t)-f(x^{-})+ f(x+t)-f(x^{+})]dt \ . \quad (3.7)$$

Thus, we take the absolute value at the above:

$$\left|A_{\{y\}}f(x)-\frac{1}{2}[f(x^{+})+f(x^{-})]\right|$$
$$=\left|\frac{1}{L}\int_{0}^{L} p_{\{y\}}(t)[f(x-t)+f(x+t)- f(x^{+})-f(x^{-})]dt\right|$$
$$\leq \frac{1}{L}\left\{\left|\int_{0}^{L} p_{\{y\}}(t)[f(x-t)-f(x^{-})]dt\right| + \left|\int_{0}^{L} p_{\{y\}}(t)[f(x+t)-f(x^{+})]dt\right|\right\}.$$

We choose one integral and the other integral is the same proof.

$$\left|\int_{0}^{L} p_{\{y\}}(t)[f(x-t)-f(x^{-})]dt\right|$$
$$\leq \int_{0}^{L}\left|p_{\{y\}}(t)[f(x-t)-f(x^{-})]\right|dt$$
$$= \int_{0}^{\delta}\left|p_{\{y\}}(t)[f(x-t)-f(x^{-})]\right|dt + \int_{\delta}^{L}\left|p_{\{y\}}(t)[f(x-t)-f(x^{-})]\right|dt.$$

Given$\varepsilon > 0$, we can choose $\delta > 0$, such that

$$\sup_{0\leq t\leq\delta}|f(x-t)-f(x^{-})| < \frac{1}{L}.\frac{\varepsilon}{2},$$

Because $f(x^{-}) = \lim_{t\to 0^{+}} f(x-t)$, consequently, we can bound the first integral:

$\int_0^{\delta}\left|p_{\{y\}}(t)[f(x-t)-f(x^-)]\right|dt \le$
$\int_0^{\delta} p_{\{y\}}(t)\sup_{0\le t\le\delta}|f(x-t)-f(x^-)|dt <$
$\int_{-L}^{L} p_{\{y\}}(t)\frac{1}{L}.\frac{\varepsilon}{2}dt = \frac{\varepsilon}{2}.$

The second integral is:

$\int_{\delta}^{L}\left|p_{\{y\}}(t)[f(x-t)-f(x^-)]\right| dt \le$
$\mathrm{Sup}_{\delta\le t\le L}\, p_{\{y\}}(t)\int_{\delta}^{L}|f(x-t)-f(x^-)|dt,$

since

$\int_{\delta}^{L}|f(x-t)-f(x^-)|dt \le \int_{\delta}^{L}|f(x-t)|dt + \int_{\delta}^{L}|f(x^-)|\,dt$
$\le \int_{-L}^{L}|f(x-t)|dt + \int_0^{L}|f(x^-)|\,dt =$
$\int_{-L}^{L}|f(t)|dt + L|f(x^-)|.$

Where the equation holds by periodicity of. Notice that the result is finite because f is integrable. Also, from the last theorem of Poisson kernel on theorem.3.1, for **r** close enough to one**.**

$$\mathrm{Sup}_{\delta\le t\le L}\, p_{\{y\}}(t) < \frac{1}{\int_{-L}^{L}|f(t)|dt+L|f(x^-)|}.\frac{\varepsilon}{2}\ .$$

Thus

$$\int_{\delta}^{L} p_{\{y\}}(t)|[f(x-t)-f(x^-)]|\,dt < \frac{\varepsilon}{2}\ .$$

Since $0 \le \{y\} < 1$, we obtain that

$$\left|\int_0^{L} p_{\{y\}}(t)[f(x-t)-f(x^-)]dt\right| < \varepsilon\,,$$

consequently

$$\int_0^{L} p_{\{y\}}(t)[f(x-t)-f(x^-)]dt \xrightarrow[n\to\infty]{} 0\ .$$

Similarly.

$$\int_0^{L} p_{\{y\}}(t)[f(x+t)-f(x^+)]\,dt \xrightarrow[n\to\infty]{} 0\ .$$

The above theorem is proven.

Remark. 2.1. If f is continuous at x, then $\lim_{\{y\}\to 1^-} A_{\{y\}}f(x) = f(x).$

Remark 3.1. If we take $u_n(y) = \{y\}^n$ in Theorem 3.1, then we get $P_{\{y\}}(t).$

The prove is the same as proof at the above theorem.

2. 3. Generalized Poisson Summability

In this section, we will work to generalize the Poisson kernel and then study the uniformly convergence it.

Theorem 3.1. Let $D_m(t)$ be a Dirichlet kernel, such that partial summation is bounded on $[-L,L]$-$(-\delta,\delta)$ and there exists a function sequence $\{u_n(x)\}$, be uniformly decreasing to zero. Then the series

$$\frac{u_0}{2} + \sum_{k=1}^{\infty} u_k(x)\cos\left(\frac{k\pi t}{L}\right),$$

be converges uniformly on $[-L,L]$-$(-\delta,\delta)$.

Proof:

From Definition 1.5. we have

$$|\,D_m(t)| = \left|\frac{1}{2} + \sum_{k=1}^{m}\cos\left(\frac{\pi k t}{L}\right)\right|.$$

Multiple by $2\sin\left(\frac{\pi t}{2L}\right)$, we obtain that

$$\left|2\sin\left(\frac{\pi t}{2L}\right) D_m(t)\right| = \left|\sin\left(\frac{\pi t}{2L}\right) + \sum_{k=1}^{m} 2\sin\left(\frac{\pi t}{2L}\right)\cos\left(\frac{\pi k t}{L}\right)\right| = \left|\sin\left(\frac{\pi t}{2L}\right) + \sum_{k=1}^{m}\left(\sin\left(\frac{1}{2}+k\right)\frac{\pi t}{L} - \sin\left(\frac{1}{2}-k\right)\frac{\pi t}{L}\right)\right|.$$

Decipher the above sums

$$|\,D_m(t)| = \left|\frac{\sin\left(m+\frac{1}{2}\right)\frac{\pi t}{L}}{2\sin\left(\frac{\pi t}{2L}\right)}\right| \le \frac{1}{2\left|\sin\left(\frac{\pi t}{2L}\right)\right|}\ ,$$

bounded on $[-L,L]$--$(-\delta,\delta)$ and $\{u_n(x)\}$, is decreasing and $u_n(x) \xrightarrow[n\to\infty]{} o$, then by (Dirichlet′s Theorem 2.3), which completes the proof.

The above theorem, gave generalized to the Poisson kernel.

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دراسة حول تمثيل Abel-Poisson Summability بناءً على دالة الجزء الكسري وتعميمها

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الملخص:
مجموع بواسون استعمل طريقة آبل من أجل دراسة التقارب المنتظم لمتسلسلة فورية. في هذا البحث نعطي بدلاً من الثابت r في طريقة آبل، أخذ دالة الجزء الكسري من خلال متسلسلة فورية على الفترة [-L,L]، حصلنا على نتائج جديدة. وأخيراً نعمم مجموع بواسون ودراسة التقارب المنتظم .

الكلمات المفتاحية:
مجموع بواسون، متسلسلة فورية ، نواة درشلية ، نواة بواسون، دالة الجزء الكسري.