JEF/Journal of Education Faculties
Volume 18, Issue (2), 2024

مجلة كليات التربية - جامعة عدن
المجلد 18 ، العدد (2)، 2024 م

Exact Solutions to New Potential Nonlinear Partial Differential Equations Using Extended Generalized Riccati Equation Mapping Method

Mubarak A. H. ALqufail[1] **S.A. AL-Damki**[2]

Department of Mathematics Faculty of Education Aden, Aden University

DOI: https://doi.org/10.47372/jef.(2024)18.2.104

Abstract: In this paper, we present a new model of Kadomtsev-Petviashvili-Benjamin-Bona-Mahony (KP-BBM) equation, namely, potential of (KP-BBM), and potential of the combined "Korteweg-de Vries and negative-order Korteweg-de Vries" with "Calogero-Bogoyavlenskii Schiff and negative-order Calogero-Bogoyavlenskii Schiff equation", namely, potential c(KdV-nKdV with CBS-nCBS). We apply the extended generalized Riccati equation mapping method to solve the new models. Exact travelling wave solutions are obtained and expressed in terms of hyperbolic functions, trigonometric functions and rational functions.

Keywords: potential (KP-BBM) equation -potential c(KdV-nKdV with CBS-nCBS) equation- Accurate solutions and the method for plotting the extended generalized Riccati equation.

Introduction: In recent years, directly searching for exact solutions of nonlinear partial differential equations (NLPDEs) has become more and more attractive field in different branches of physics and applied mathematics. These equations appear in condensed matter ,solid state physics, fluid mechanics, chemical kinetics, plasma physics, nonlinear optics, propagation of fluxions in Josephson junctions, theory of turbulence, ocean dynamics, biophysics and star formation and many others.

In fact the computer symbolic system such as maple or mathematica allow us to preform complicated and tedious calculations. Many algorithms have been proposed to find exact solutions for (NLPDEs) using these programs.

In order to get exact solutions directly, many powerful methods have been introduced such as the Hirota's direct method [3,11], $(G'/G, 1/G)$ expansion method [1], the tanh- coth method [16], the Jacobi elliptic function expansion method [6],

A Table lookup method [10], the exp(-φ(η))-expansion method [2], the Bäcklund transformation method [8], inverse scattering method [5], mapping method [12], proposed method [14] and homogenous balance method [19].

Recently, Shun-dong Zhu [20], introduced a new approach, namely, the extended generalized Riccati equation mapping method, for a reliable treatment of the nonlinear wave equations. The useful extended generalized Riccati equation mapping method is then widely used by many authors [4,7,9,13].

Description of the Extended Generalized Riccati Equation Mapping Method

Consider the general nonlinear partial differential equation (NLPDE), say, in two variables

$$p(u, u_x, u_t, u_{xx}, u_{xt}, \dots) = 0. \qquad (1)$$

Eq. (1) can be solved by using the following steps:

Step 1: Use the wave variable $\xi = \mu(x - ct)$, where μ is the wave number and c is the wave speed to change the Eq.(1) in to

$$Q(u, u', u'', \dots) = 0\,, \qquad (2)$$

where Eq. (2) is the nonlinear ordinary differential equation (NLODE) and $'$ denotes to the differentiation with respect to ξ.

Step 2: We suppose that the solution of Eq. (2) has the form

$$u(x,t) = u(\xi) = \sum_{i=-n}^{n} a_i (Q(\xi))^i \quad , \qquad (3)$$

where the coefficients $a_i (i = 0, 1, 2, \dots, n)$, are constants to be determined later, and Q = Q(ξ) satisfies a nonlinear ordinary differential equation

$$Q'(\xi) = r + pQ(\xi) + qQ^2(\xi)\ , \qquad (4)$$

where p , q and r are constants to be determined later .

The value of positive integer *n* is easy to find by balancing the highest order nonlinear terms with the highest order derivative term appearing in Eq. (2).

Step 3: Substituting Eq. (3) along with Eq.(4) into Eq. (2) and collecting all the coefficients of $Q^i(\xi)$, $(i = 0, 1, 2, \dots, n)$, then setting them to zero, yield a set of algebraic equation Solutions to the resulting algebraic system are derived by using the extended generalized Riccati equation mapping method with the aid of Maple.

Step 4: The solutions of Eq.(4), can be divided into four different families as follows :

Family 1: When $p^2 - 4qr < 0$ and $pq \neq 0$ or $qr \neq 0$, the solutions of Eq. (4) are

1. $G(\xi) = \frac{1}{2q}\left(-P + \sqrt{4qr - P^2}\, tan\left(\frac{\sqrt{4qr-P^2}}{2}\xi\right)\right)$,

2. $G(\xi) = \frac{-1}{2q}\left(P + \sqrt{4qr - P^2}\, cot\left(\frac{\sqrt{4qr-P^2}}{2}\xi\right)\right)$,

3. $G(\xi) = \frac{1}{2q}\left(-P + \sqrt{4qr - P^2}\ \left(tan(\sqrt{4qr - P^2}\xi) \pm\ sec(\sqrt{4qr - P^2}\xi)\right)\right)$,

4. $G(\xi) = \frac{-1}{2q}\left(P + \sqrt{4qr - P^2}\ \left(cot(\sqrt{4qr - P^2}\xi) \pm csc(\sqrt{4qr - P^2}\xi)\right)\right)$,

5. $G(\xi) = \frac{1}{4q}\left(-2P + \sqrt{4qr - P^2}\ \left(tan\left(\frac{\sqrt{4qr-P^2}}{4}\xi\right) - cot\left(\frac{\sqrt{4qr-P^2}}{4}\xi\right)\right)\right)$,

6. $G(\xi) = \frac{1}{2q}\left(-P + \frac{\sqrt{(4qr-P^2)(A^2-B^2)} - A\sqrt{4qr-P^2}\, cos\left(\sqrt{4qr-P^2}\xi\right)}{A\, sin\left(\sqrt{4qr-P^2}\xi\right) + B}\right)$,

7. $G(\xi)=\frac{1}{2q}\left(-P-\frac{\sqrt{(4qr-P^2)(A^2-B^2)}+A\sqrt{4qr-P^2}\cos\left(\sqrt{4qr-P^2}\xi\right)}{A\sin\left(\sqrt{4qr-P^2}\xi\right)+B}\right),$

where A and B are two non-zero real constants satisfies $A^2 - B^2 > 0$.

8. $G(\xi)=\frac{-2r\cos\left(\frac{\sqrt{4qr-P^2}}{2}\xi\right)}{\sqrt{4qr-P^2}\sin\left(\frac{\sqrt{4qr-P^2}}{2}\xi\right)+p\cos\left(\frac{\sqrt{4qr-P^2}}{2}\xi\right)},$

9. $G(\xi)=\frac{2r\sin\left(\frac{\sqrt{4qr-P^2}}{2}\xi\right)}{-p\sin\left(\frac{\sqrt{4qr-P^2}}{2}\xi\right)+\sqrt{4qr-P^2}\cos\left(\frac{\sqrt{4qr-P^2}}{2}\xi\right)},$

10. $G(\xi)=\frac{-2r\cos\left(\sqrt{4qr-P^2}\xi\right)}{\sqrt{4qr-P^2}\sin\left(\sqrt{4qr-P^2}\xi\right)+p\cos\left(\sqrt{4qr-P^2}\xi\right)\pm\sqrt{4qr-P^2}},$

11. $G(\xi)=\frac{2r\sin\left(\sqrt{4qr-P^2}\xi\right)}{-p\sin\left(\sqrt{4qr-P^2}\xi\right)+\sqrt{4qr-P^2}\cos\left(\sqrt{4qr-P^2}\xi\right)\pm\sqrt{4qr-P^2}},$

12. $G(\xi)=\frac{4r\sin\left(\frac{\sqrt{4qr-P^2}}{4}\xi\right)\cos\left(\frac{\sqrt{4qr-P^2}}{4}\xi\right)}{-2p\sin\left(\frac{\sqrt{4qr-P^2}}{4}\xi\right)\cos\left(\frac{\sqrt{4qr-P^2}}{4}\xi\right)+2\sqrt{4qr-P^2}\cos^2\left(\frac{\sqrt{4qr-P^2}}{4}\xi\right)-\sqrt{4qr-P^2}}.$

Family 2: When $p^2-4qr>0$ and $pq\neq 0$ or $qr\neq 0$, the solutions of Eq. (4) are

13. $G(\xi)=\frac{-1}{2q}\left(P+\sqrt{P^2-4qr}\tanh\left(\frac{\sqrt{P^2-4qr}}{2}\xi\right)\right),$

14. $G(\xi)=\frac{-1}{2q}\left(P+\sqrt{P^2-4qr}\coth\left(\frac{\sqrt{P^2-4qr}}{2}\xi\right)\right),$

15. $G(\xi)=\frac{-1}{2q}\left(P+\sqrt{P^2-4qr}\left(\tanh\left(\sqrt{P^2-4qr}\xi\right)\pm i\,\mathrm{sech}\left(\sqrt{P^2-4qr}\xi\right)\right)\right),$

16. $G(\xi)=\frac{-1}{2q}\left(P+\sqrt{P^2-4qr}\left(\coth\left(\sqrt{P^2-4qr}\xi\right)\pm \mathrm{csch}\left(\sqrt{P^2-4qr}\xi\right)\right)\right),$

17. $G(\xi)=\frac{-1}{4q}\left(2P+\sqrt{P^2-4qr}\left(\tanh\left(\frac{\sqrt{P^2-4qr}}{4}\xi\right)\pm\coth\left(\frac{\sqrt{P^2-4qr}}{4}\xi\right)\right)\right),$

18. $G(\xi)=\frac{1}{2q}\left(-P+\frac{\sqrt{(P^2-4qr)(A^2+B^2)}-A\sqrt{P^2-4qr}\cosh\left(\sqrt{P^2-4qr}\xi\right)}{A\sinh\left(\sqrt{P^2-4qr}\xi\right)+B}\right),$

19. $G(\xi)=\frac{1}{2q}\left(-P-\frac{\sqrt{(P^2-4qr)(B^2-A^2)}+A\sqrt{P^2-4qr}\sinh\left(\sqrt{P^2-4qr}\xi\right)}{A\cosh\left(\sqrt{P^2-4qr}\xi\right)+B}\right),$

where A and B are two non-zero real constants and satisfies $B^2 - A^2 > 0$,

20. $G(\xi)=\dfrac{2r\cosh\left(\frac{\sqrt{P^2-4qr}}{2}\xi\right)}{\sqrt{P^2-4qr}\sinh\left(\frac{\sqrt{P^2-4qr}}{2}\xi\right)-p\cosh\left(\frac{\sqrt{P^2-4qr}}{2}\xi\right)}$,

21. $G(\xi)=\dfrac{-2r\sinh\left(\frac{\sqrt{P^2-4qr}}{2}\xi\right)}{p\sinh\left(\frac{\sqrt{P^2-4qr}}{2}\xi\right)-\sqrt{P^2-4qr}\cosh\left(\frac{\sqrt{P^2-4qr}}{2}\xi\right)}$,

22. $G(\xi)=\dfrac{2r\cosh\left(\sqrt{P^2-4qr}\xi\right)}{\sqrt{P^2-4qr}\sinh\left(\sqrt{P^2-4qr}\xi\right)-p\cosh\left(\sqrt{P^2-4qr}\xi\right)\pm i\sqrt{P^2-4qr}}$, $i=\sqrt{-1}$

23. $G(\xi)=\dfrac{2r\sinh\left(\sqrt{P^2-4qr}\xi\right)}{-p\sinh\left(\sqrt{P^2-4qr}\xi\right)+\sqrt{P^2-4qr}\cosh\left(\sqrt{P^2-4qr}\xi\right)\pm\sqrt{P^2-4qr}}$,

24. $G(\xi)=\dfrac{4r\sinh\left(\frac{\sqrt{P^2-4qr}}{4}\xi\right)\cosh\left(\frac{\sqrt{P^2-4qr}}{4}\xi\right)}{-2p\sinh\left(\frac{\sqrt{P^2-4qr}}{4}\xi\right)\cosh\left(\frac{\sqrt{P^2-4qr}}{4}\xi\right)+2\sqrt{P^2-4qr}\cosh^2\left(\frac{\sqrt{P^2-4qr}}{4}\xi\right)-\sqrt{P^2-4qr}}$.

Family 3: When $r=0$ and $pq\neq 0$, the solutions of Eq. (4) becomes:

25. $G(\xi)=\dfrac{-p\,k}{q\,(k+\cosh(p\xi)-\sinh(p\xi))}$,

26. $G(\xi)=\dfrac{-p\,(\cosh(p\xi)+\sinh(p\xi))}{q\,(k+\cosh(p\xi)+\sinh(p\xi))}$.

where k is an arbitrary constant.

Family 4: When $r=p=0$ and $q\neq 0$, the solutions of Eq. (4) becomes:

27. $G(\xi)=\dfrac{-1}{q\,\xi+l}$.

where l is an arbitrary constant.

The multiple exact special solutions of nonlinear partial differential equation (1) are obtained by making use of Eq. (3) and the solutions of Eq. (4).

Exact Solutions for Potential c(KdV-nKdV with CBS-nCBS) Equation

We consider a combined Korteweg–de Vries and negative-order Korteweg–de Vries with Calogero-Bogoyavlenskii Schiff and negative-order Calogero-Bogoyavlenskii Schiff equation c(KdV-nKdV with CBS-nCBS) [17,18] as the form

$v_t+v_{xxy}+v_{xxt}+v_{xxx}+4v(v_y+v_t)+2v_x\partial_x^{-1}(v_y+v_t)+6vv_x=0, v=v(x,y,t)$, (5), where

$$v_t+v_{xxx}+v_{xxt}+6vv_x+4vv_t+2v_x\partial_x^{-1}v_t=0, \qquad (6)$$

is the Korteweg–de Vries and negative-order Korteweg–de Vries (KdV-nKdV) equation [21],

and $v_t+v_{xxt}+v_{xxy}+4vv_t+4vv_y+2v_x\partial_x^{-1}(v_y+v_t)=0,$ (7)

is the Calogero-Bogoyavlenskii Schiff and negative-order Calogero-Bogoyavlenskii Schiff (CBS-nCBS) equation [22].

The assumption $v(x,y,t)=u_x(x,y,t)$ transformed Eq. (5) to

$$u_{xt}+u_{xxxy}+u_{xxxt}+u_{xxxx}+4u_x(u_{xy}+u_{xt})+2u_{xx}(u_y+u_t)+6u_xu_{xx}=0, \quad (8)$$

Eq. (8) is called a potential combined Korteweg–de Vries and negative-order Korteweg–de Vries with Calogero-Bogoyavlenskii Schiff and negative-order Calogero-Bogoyavlenskii Schiff equation of Eq. (5) and denoted by potential c(KdV-nKdV with CBS-nCBS). substituting $u(x,y,t)=u(\xi)$, $\xi=\lambda(x+y-ct)$ in Eq. (8) and integrating the resulting equation, we find

$$-cu'(\xi)+\lambda^2(2-c)u'''(\xi)+3\lambda(2-c)\left(u'(\xi)\right)^2=0. \quad (9)$$

Eq. (9) is nonlinear ordinary differential equation.

Balancing the highest order of the nonlinear term $(u')^2$ with the highest order derivative u''' gives $2n+2=n+3$ that gives $n=1$. Now, we apply the extended generalized Riccati equation mapping method to solve our equation.

Consequently, we get the original solutions as the follows:

Assume, the solution of Eq. (9) has the form

$$u(\xi)=a_0+a_1\,Q(\xi), \quad (10)$$

where a_0 and a_1 are constants.

By substituting Eq. (10) in Eq. (9) and using Eq. (4), the left hand side is converted into polynomials in $Q^i(\xi), 0\le i\le 4$, setting each coefficient of these resulted polynomials to zero, we obtain a set of algebraic equations for a_0, a_1 , c, p, q , r and λ. Solving the resulting system of algebraic equations with help Maple, we obtain

Case 1: $a_0=a_0\,, a_1=-2q\lambda, p=0, q=q, r=r, \lambda=\lambda, c=\frac{8rq\lambda^2}{4rq\lambda^2-1}$.

Case 2: $a_0=a_0\,, a_1=-\frac{2q\sqrt{\frac{-c}{c-2}}}{p}, p=p, q=q, r=0, \lambda=\frac{\sqrt{\frac{-c}{c-2}}}{p}, c=c$.

Case 3: $a_0=a_0\,, a_1=\frac{2q\sqrt{\frac{-c}{c-2}}}{p}, p=p, q=q, r=0, \lambda=-\frac{\sqrt{\frac{-c}{c-2}}}{p}, c=c$.

The above cases of values yields the following exact solutions of potential c(KdV-nKdV and CBS-nCBS) equation using Eq.(10) and the solutions of Eq.(4) .

Exact traveling wave solution of Eq. (8) for **Case 1** given by the following:

Family 1: $p^2-4qr<0$ and $pq\ne 0$ or $qr\ne 0$ are

$$u_1\,(x,y,t)=a_0-\lambda\left(2\sqrt{qr}\,tan\left(\sqrt{qr}\xi\right)\right), \text{where } \xi=\lambda\left(x+y-\frac{8qr\lambda^2}{4qr\lambda^2-1}t\right),$$

$$u_2(x,y,t)=a_0+\lambda\left(2\sqrt{qr}\,cot\left(\sqrt{qr}\xi\right)\right),$$

$$u_{3,4}(x,y,t)=a_0-\lambda\left(2\sqrt{qr}\left(tan\left(2\sqrt{qr}\xi\right)\pm sec\left(2\sqrt{qr}\xi\right)\right)\right),$$

$$u_{5,6}(x,y,t)=a_0+\lambda\left(2\sqrt{qr}\left(cot\left(2\sqrt{qr}\xi\right)\pm csc\left(2\sqrt{qr}\xi\right)\right)\right),$$

$$u_7(x,y,t)=a_0-\frac{\lambda}{2}\left(2\sqrt{qr}\left(tan\left(\frac{\sqrt{qr}}{2}\xi\right)-cot\left(\frac{\sqrt{qr}}{2}\xi\right)\right)\right),$$

$$u_8(x,y,t) = a_0 - \lambda\left(\frac{2\sqrt{qr}(\sqrt{(A^2-B^2)}-A\cos(2\sqrt{qr}\xi))}{A\sin(2\sqrt{qr}\xi)+B}\right),$$

$$u_9(x,y,t) = a_0 + \lambda\left(\frac{2\sqrt{qr}(\sqrt{(A^2-B^2)}+A\cos(2\sqrt{qr}\xi))}{A\sin(2\sqrt{qr}\xi)+B}\right),$$

where A and B are two non-zero real constant and satisfies $A^2 - B^2 > 0$.

$$u_{10}(x,y,t) = a_0 - 2q\lambda\left(\frac{-2\,r\cos(\sqrt{qr}\xi)}{2\sqrt{rq}\sin(\sqrt{qr}\xi)}\right),$$

$$u_{11}(x,y,t) = a_0 - 2q\lambda\left(\frac{2r\sin(\sqrt{qr}\xi)}{2\sqrt{rq}\cos(\sqrt{qr}\xi)}\right),$$

$$u_{12,13}(x,y,t) = a_0 - 2q\lambda\left(\frac{-2r\cos(2\sqrt{qr}\xi)}{2\sqrt{rq}\sin(2\sqrt{qr}\xi)\pm 1}\right),$$

$$u_{14,15}(x,y,t) = a_0 - 2q\lambda\left(\frac{2r\sin(2\sqrt{qr}\xi)}{2\sqrt{qr}\cos(2\sqrt{qr}\xi)\pm 1}\right),$$

$$u_{16}(x,y,t) = a_0 - 2q\lambda\left(\frac{4\,r\sin\left(\frac{\sqrt{qr}}{2}\xi\right)\cos\left(\frac{\sqrt{qr}}{2}\xi\right)}{2\sqrt{qr}\left(2\cos^2\left(\frac{\sqrt{qr}}{2}\xi\right)-1\right)}\right).$$

Family 2: $p^2 - 4qr > 0$ and $pq \neq 0$ or $qr \neq 0$ are

$$u_{17}(x,y,t) = a_0 + \lambda\left(2\sqrt{-qr}\tanh(\sqrt{-qr}\xi)\right), \text{ where } \xi = \lambda\left(x + y - \frac{8qr\lambda^2}{4qr\lambda^2-1}t\right),$$

$$u_{18}(x,y,t) = a_0 + \lambda\left(2\sqrt{-qr}\coth(\sqrt{-qr}\xi)\right),$$

$$u_{19,20}(x,y,t) = a_0 + \lambda\left(2\sqrt{-qr}\left(\tanh(2\sqrt{-qr}\xi) \pm i\,\mathrm{sech}(2\sqrt{-qr}\xi)\right)\right),$$

$$u_{21,22}(x,y,t) = a_0 + \lambda\left(2\sqrt{-qr}\left(\coth(2\sqrt{-qr}\xi) \pm \mathrm{csch}(2\sqrt{-qr}\xi)\right)\right),$$

$$u_{23}(x,y,t) = a_0 + \frac{\lambda}{2}\left(2\sqrt{-qr}\left(\tanh\left(\frac{\sqrt{-qr}}{2}\xi\right) + \coth\left(\frac{\sqrt{-qr}}{2}\xi\right)\right)\right),$$

$$u_{24}(x,y,t) = a_0 - \lambda\left(\frac{2\sqrt{-qr}\sqrt{(A^2+B^2)}-A\cosh(2\sqrt{-qr}\xi)}{A\sinh(2\sqrt{-qr}\xi)+B}\right),$$

$$u_{25}(x,y,t) = a_0 + \lambda\left(\frac{2\sqrt{-qr}\sqrt{(B^2-A^2)}+A\sinh(2\sqrt{-qr}\xi)}{A\cosh(2\sqrt{-qr}\xi)+B}\right),$$

where A and B are two non-zero real constant and satisfies $B^2 - A^2 > 0$.

$$u_{26}(x,y,t) = a_0 - 2q\lambda\left(\frac{2r\cosh(2\sqrt{-qr}\xi)}{2\sqrt{-qr}\sinh(2\sqrt{-qr}\xi)}\right),$$

$$u_{27}(x,y,t) = a_0 - 2q\lambda\left(\frac{-2r\sinh(2\sqrt{-qr}\xi)}{2\sqrt{-qr}\cosh(2\sqrt{-qr}\xi)}\right),$$

$$u_{28,29}(x,y,t) = a_0 - 2q\lambda\left(\frac{2\,r\cosh(2\sqrt{-qr}\xi)}{2\sqrt{-qr}\sinh(2\sqrt{-qr}\xi)\pm i}\right),$$

$$u_{30,31}(x,y,t) = a_0 - 2q\lambda\left(\frac{2\,r\sinh(2\sqrt{-qr}\xi)}{2\sqrt{-qr}\cosh(2\sqrt{-qr}\xi)\pm 1}\right),$$

$$u_{32}(x,y,t) = a_0 - 2q\lambda\left(\frac{4\,r\sinh\left(\frac{\sqrt{-qr}}{2}\xi\right)\cosh\left(\frac{\sqrt{-qr}}{2}\xi\right)}{2\sqrt{-qr}\left(2\cosh^2\left(\frac{\sqrt{-qr}}{2}\xi\right)-1\right)}\right).$$

Exact traveling wave solution of Eq. (8) for **Case 2** given by the following:

Family 1: $p^2 - 4qr < 0$ and $pq \neq 0$ or $qr \neq 0$:

$p^2 - 4qr \nless 0$ $\; since\; r = 0\,, given\; that\; (p^2, p \in R) < 0$ means that is not true, i. e no solution in this family.

Family 2: $p^2 - 4qr > 0$ and $pq \neq 0$ or $qr \neq 0$ are:

$$u_{33}(x,y,t) = a_0 + \varphi\left(1 + \tanh\left(\frac{\varphi}{2}\,\eta\right)\right),$$

where $\eta = x + y - ct,\ \varphi = \sqrt{\frac{-c}{c-2}}$, and c is arbitrary constant.

$$u_{34}(x,y,t) = a_0 + \varphi\left(1 + \coth\left(\frac{\varphi}{2}\,\eta\right)\right),$$

$$u_{35,36}(x,y,t) = a_0 + \varphi\,(1 + \tanh(\varphi\ \eta) \pm i\,sech(\varphi\ \eta)),$$

$$u_{37,38}(x,y,t) = a_0 + \varphi\,(1 + \coth(\varphi\ \eta) \pm csch(\varphi\,\eta)),$$

$$u_{39}(x,y,t) = a_0 + \frac{\varphi}{2}\left(2 + \tanh\left(\frac{\varphi}{4}\,\eta\right) + \coth\left(\frac{\varphi}{4}\,\eta\right)\right)$$

$$u_{40}(x,y,t) = a_0 - \varphi\left(-1 + \frac{\sqrt{A^2+B^2} - A\cosh(\varphi\ \eta)}{A\sinh(\varphi\ \xi)+B}\right),$$

$$u_{41}(x,y,t) = a_0 - \varphi\left(-1 - \frac{\sqrt{B^2-A^2} + A\sinh(\varphi\ \eta)}{A\cosh(\varphi\ \eta)+B}\right).$$

where A and B are two non-zero real constant and satisfies $B^2 - A^2 > 0$. For $\frac{-c}{c-2} < 0$, we get

$$u_{42}(x,y,t) = a_0 - \mu\left(-i - \tan\left(-\frac{\mu}{2}\,\eta\right)\right), \text{ where } \eta = x + y - ct,\ \mu = \sqrt{\frac{c}{c-2}}\ .$$

$$u_{43}(x,y,t) = a_0 + \mu\left(i - \cot\left(-\frac{\mu}{2}\,\eta\right)\right),$$

$$u_{44,45}(x,y,t) = a_0 - \mu\,(-i - \tan(-\mu\ \eta) \pm \sec(-\mu\ \eta)),$$

$$u_{46,47}(x,y,t) = a_0 + \mu\,(i - \cot(-\mu\ \eta) \pm \csc(-\mu\ \eta)),$$

$$u_{48}(x,y,t) = a_0 - \frac{\mu}{2}\left(-2i - \tan\left(-\frac{\mu}{4}\,\eta\right) + \cot\left(-\frac{\mu}{4}\,\eta\right)\right),$$

$$u_{49}(x,y,t) = a_0 - \mu\left(-i - \frac{\sqrt{A^2-B^2} - A\cos(-\mu\ \eta)}{A\sin(-\mu\ \eta)+B}\right),$$

$$u_{50}(x,y,t) = a_0 - \mu\left(-i + \frac{\sqrt{A^2-B^2} + A\cos(-\mu\ \eta)}{A\sin(-\mu\ \eta)+B}\right).$$

Where A and B are two non-zero real constant and satisfies $A^2 - B^2 > 0$.

Family 3: When $r = 0$ and $qp \neq 0$ are:

$$u_{51}(x,y,t) = a_0 + \frac{2k\varphi}{k+\cosh(\varphi\ \eta) - \sinh(\varphi\ \eta)},$$

$$u_{52}(x,y,t) = a_0 + \frac{2\,\varphi(\cosh(\varphi\ \eta) + \sinh(\varphi\ \eta))}{k+\cosh(\varphi\ \eta) + \sinh(\varphi\ \eta)}\ .$$

where $\eta = x + y - ct,\ \varphi = \sqrt{\frac{-c}{c-2}}$ and c is arbitrary constant.

Exact traveling wave solution of Eq. (8) for **Case 3** given by the following:

Family 1: $p^2 - 4qr < 0$ and $pq \neq 0$ or $qr \neq 0$:

$p^2 - 4qr \nless 0\ \ since\ r = 0\,, given\ that\ (p^2, p \in R) < 0$ means that is not true, i. e no solution in this family.

Family 2: $p^2 - 4qr > 0$ and $pq \neq 0$ or $qr \neq 0$ are:

$$u_{53}(x,y,t) = a_0 - \varphi\left(1 + \tanh\left(-\frac{\varphi}{2}\,\eta\right)\right),$$

where $\eta = x + y - ct,\ \varphi = \sqrt{\frac{-c}{c-2}}$ and c is arbitrary constant.

$$u_{54}(x,y,t) = a_0 - \varphi\left(1 + \coth\left(-\frac{\varphi}{2}\,\eta\right)\right),$$

$$u_{55,56}(x,y,t) = a_0 - \varphi\,(1 + \tanh(-\varphi\ \eta) \pm i\,sech(-\varphi\ \eta)),$$

$$u_{57,58}(x,y,t) = a_0 - \varphi\left(1 + \coth(-\varphi\ \eta) \pm \operatorname{csch}(-\varphi\ \eta)\right),$$
$$u_{59}(x,y,t) = a_0 - \frac{\varphi}{2}\left(2 + \tanh\left(-\frac{\varphi}{4}\eta\right) + \coth\left(-\frac{\varphi}{4}\eta\right)\right)$$
$$u_{60}(x,y,t) = a_0 + \varphi\left(-1 + \frac{\sqrt{A^2+B^2} - A\cosh(-\varphi\ \eta)}{A\sinh(-\varphi\ \eta)+B}\right),$$
$$u_{61}(x,y,t) = a_0 + \varphi\left(-1 - \frac{\sqrt{B^2-A^2} + A\sinh(-\varphi\ \eta)}{A\cosh(-\varphi\ \eta)+B}\right).$$

where A and B are two non-zero real constant and satisfies $B^2 - A^2 > 0$. For $\frac{-c}{c-2} < 0$, we get

$$u_{62}(x,y,t) = a_0 + \mu\left(-i - \tan\left(\frac{\mu}{2}\eta\right)\right), \text{ where } \eta = x + y - ct,\ \mu = \sqrt{\frac{c}{c-2}}\ .$$
$$u_{63}(x,y,t) = a_0 - \mu\left(i - \cot\left(\frac{\mu}{2}\eta\right)\right),$$
$$u_{64,65}(x,y,t) = a_0 + \mu\left(-i - \tan(\mu\ \eta) \pm \sec(\mu\ \eta)\right),$$
$$u_{66,67}(x,y,t) = a_0 - \mu\left(i - \cot(\mu\ \eta) \pm \csc(\mu\ \eta)\right),$$
$$u_{68}(x,y,t) = a_0 + \frac{\mu}{2}\left(-2i - \tan\left(\frac{\mu}{4}\eta\right) + \cot\left(\frac{\mu}{4}\eta\right)\right),$$
$$u_{69}(x,y,t) = a_0 + \mu\left(-i - \frac{\sqrt{A^2-B^2} - A\cos(\mu\ \eta)}{A\sin(\mu\ \eta)+B}\right),$$
$$u_{70}(x,y,t) = a_0 + \mu\left(-i + \frac{\sqrt{A^2-B^2} + A\cos(\mu\ \eta)}{A\sin(\mu\ \eta)+B}\right).$$

where A and B are two non-zero real constant and satisfies $A^2 - B^2 > 0$.

Family 3: When $r = 0$ and $qp \neq 0$ are:

$$u_{71}(x,y,t) = a_0 - \frac{2k\varphi}{k + \cosh(-\varphi\ \eta) - \sinh(-\varphi\ \eta)},$$
$$u_{72}(x,y,t) = a_0 - \frac{2\varphi\left(\cosh(-\varphi\ \eta) + \sinh(-\varphi\ \eta)\right)}{k + \cosh(-\varphi\ \eta) + \sinh(-\varphi\ \eta)}\ .$$

where $\eta = x + y - ct,\ \varphi = \sqrt{\frac{-c}{c-2}}$ and c is arbitrary constant.

Exact Solutions for potential (KP-BBM) Equation

We consider a Kadomtsev Petviashvili-Benjamin-Bona-Mahony equation [15] as the form:

$$u_t + u_x + 2\alpha u u_x - \beta u_{xxt} + \gamma\, \partial_x^{-1} u_{yy} = 0, u = u(x,y,t). \tag{11}$$

with α, β and γ being arbitrary nonzero-constants.

The assumption $v(x,y,t) = u_x(x,y,t)$ transformed Eq. (11) to

$$v_{xt} + v_{xx} + 2\alpha v_x v_{xx} + \beta v_{xxxt} + \gamma v_{yy} = 0\ , \tag{12}$$

Eq. (12) is called a potential Kadomtsev Petviashvili-Benjamin-Bona-Mahony equation of Eq.(11) and denoted by p(KP-BBM). Substituting $v(x,y,t) = v(\xi)$, $\xi = \lambda(x + y - \omega t)$ in Eq. (12) and integrating the resulting equation, we find

$$\beta\omega\lambda^2 v'''(\xi) + \alpha\lambda\left(v'(\xi)\right)^2 + (1 + \gamma - \omega)v'(\xi) = 0, \tag{13}$$

Eq. (13) is nonlinear ordinary differential equation. Balancing the highest order of the nonlinear term $(v')^2$ with the highest order derivative v''' gives $2n + 2 = n + 3$ that gives $n = 1$. Now, we apply the extended generalized Riccati equation mapping method to solve our equation. Consequently, we get the original solutions as the follows:

Assume, the solution of Eq. (13) has the form

$$v(\xi) = a_0 + a_1\, Q(\xi), \tag{14}$$

By substituting Eq. (14) in Eq. (13) and using Eq. (4),the left hand side is converted into polynomials in $Q^i(\xi), 0 \le i \le 4$, setting each coefficient of these resulted polynomials to zero, we obtain a set of algebraic equations for a_0, a_1 , ω, p, q, r and λ. Solving the resulting system of algebraic equations with help Maple, we obtain

Case 1: $a_0 = a_0\,, a_1 = -\frac{6q\beta\lambda\omega}{\alpha}, p = 0, q = q, r = -\frac{1+\gamma-\omega}{4\beta\omega\lambda^2 q}, \omega = \omega, \lambda = \lambda,$

Case 2: $a_0 = a_0\,, a_1 = \frac{6(1+\gamma)q\beta\lambda}{\alpha(\beta\lambda^2p^2-1)}, p = p, q = q, r = 0, \omega = -\frac{1+\gamma}{\beta\lambda^2p^2-1}, \lambda = \lambda,$

Case 3: $a_0 = a_0\,, a_1 = -\frac{3(1+\gamma-\omega+\beta\omega\lambda^2p^2)}{2\alpha r\lambda}, p = p, q = \frac{1+\gamma-\omega+\beta\omega\lambda^2p^2}{4\omega\beta\lambda^2 r},\ r = r,\ \omega = \omega, \lambda = \lambda,$

The above cases of values yields the following exact solutions of potential (KP-BBM) equation using Eq.(14), and the solutions of Eq.(4) .

Exact traveling wave solution of Eq.(12) for **Case 1** given by the following:

Family 1: $p^2 - 4qr < 0$ and $pq \neq 0$ or $qr \neq 0$ are:

$$v_1(x,y,t) = a_0 - \frac{3\beta\omega}{\alpha}\left(\sqrt{\Delta}\, tan\left(\frac{\sqrt{\Delta}}{2}\eta\right)\right), \text{ where } \eta = (x + y - \omega t),\ \Delta = \frac{1+\gamma-\omega}{\beta\omega}.$$

$$v_2(x,y,t) = a_0 + \frac{3\beta\omega}{\alpha}\left(\sqrt{\Delta}\, cot\left(\frac{\sqrt{\Delta}}{2}\eta\right)\right),$$

$$v_{3,4}(x,y,t) = a_0 - \frac{3\beta\omega}{\alpha}\left(\sqrt{\Delta}\,\left(tan(\sqrt{\Delta}\,\eta) \pm sec(\sqrt{\Delta}\,\eta)\right)\right),$$

$$v_{5,6}(x,y,t) = a_0 + \frac{3\beta\omega}{\alpha}\left(\sqrt{\Delta}\,\left(cot(\sqrt{\Delta}\,\eta) \pm csc(\sqrt{\Delta}\,\eta)\right)\right),$$

$$v_7(x,y,t) = a_0 - \frac{3\beta\omega}{2\alpha}\left(\sqrt{\Delta}\,\left(tan\left(\frac{\sqrt{\Delta}}{4}\eta\right) - cot\left(\frac{\sqrt{\Delta}}{4}\eta\right)\right)\right),$$

$$v_8(x,y,t) = a_0 - \frac{3\beta\omega}{\alpha}\left(\frac{\sqrt{\Delta(A^2-B^2)} - A\sqrt{\Delta}\, cos(\sqrt{\Delta}\,\eta)}{A\, sin(\sqrt{\Delta}\,\eta) + B}\right),$$

$$v_9(x,y,t) = a_0 + \frac{3\beta\omega}{\alpha}\left(\frac{\sqrt{\Delta(A^2-B^2)} + A\sqrt{\Delta}\, cos(\sqrt{\Delta}\,\eta)}{A\, sin(\sqrt{\Delta}\,\eta) + B}\right),$$

where A and B are two non-zero real constant and satisfies $A^2 - B^2 > 0$.

$$v_{10}(x,y,t) = a_0 + \frac{3\sqrt{\beta\omega}}{\alpha}\left(\frac{\sqrt{1+\gamma-\omega}\, cos\left(\frac{\sqrt{\Delta}}{2}\eta\right)}{sin\left(\frac{\sqrt{\Delta}}{2}\eta\right)}\right),$$

$$v_{11}(x,y,t) = a_0 - \frac{3\sqrt{\beta\omega}}{\alpha}\left(\frac{\sqrt{1+\gamma-\omega}\, sin\left(\frac{\sqrt{\Delta}}{2}\eta\right)}{cos\left(\frac{\sqrt{\Delta}}{2}\eta\right)}\right),$$

$$v_{12,13}(x,y,t) = a_0 + \frac{3\sqrt{\beta\omega}}{\alpha}\left(\frac{\sqrt{1+\gamma-\omega}\, cos\left(\frac{\sqrt{\Delta}}{2}\eta\right)}{sin\left(\frac{\sqrt{\Delta}}{2}\eta\right) \pm 1}\right),$$

$$v_{14,15}(x,y,t) = a_0 - \frac{3\sqrt{\beta\omega}}{\alpha}\left(\frac{\sqrt{1+\gamma-\omega}\, sin\left(\frac{\sqrt{\Delta}}{2}\eta\right)}{cos\left(\frac{\sqrt{\Delta}}{2}\eta\right) \pm 1}\right),$$

$$v_{16}(x,y,t) = a_0 - \frac{6\sqrt{\beta\omega}}{\alpha}\left(\frac{\sqrt{1+\gamma-\omega}\, sin\left(\frac{\sqrt{\Delta}}{4}\eta\right) cos\left(\frac{\sqrt{\Delta}}{4}\eta\right)}{2cos^2\left(\frac{\sqrt{\Delta}}{4}\eta\right) - 1}\right),$$

Family 2: When $p^2-4qr>0$ and $pq\neq 0$ or $qr\neq 0$ are:

we get the same solution as $[v_1,\dots,v_7]$, *and*

$$v_{17}(x,y,t)=a_0-\frac{3i\beta\omega}{\alpha}\left(\frac{\sqrt{\varphi(A^2+B^2)}-A\sqrt{\varphi}\cosh(\sqrt{\varphi}\eta)}{A\sinh(\sqrt{\varphi}\eta)+B}\right),$$

$v_{18}(x,y,t)=a_0+\frac{3i\beta\omega}{\alpha}\left(\frac{\sqrt{\varphi(B^2-A^2)}+A\sqrt{\varphi}\sinh(\sqrt{\varphi}\eta)}{A\cosh(\sqrt{\varphi}\eta)+B}\right)$, where $\eta=(x+y-\omega t),\varphi=-\frac{1+\gamma-\omega}{\beta\omega}$.

where A and B are two non-zero real constant and satisfies $B^2-A^2>0$.

we get the same solution as $[v_{10},\dots,v_{16}]$.

Exact traveling wave solution of Eq.(12) for **Case 2** given by the following:

Family 1: $p^2-4qr<0$ and $pq\neq 0$ or $qr\neq 0$:

$p^2-4qr\nless 0$ $since\ r=0,given\ that(p^2,p\in R)<0$ means that is not true, i. e no solution in this family.

Family 2: $p^2-4qr>0$ and $pq\neq 0$ or $qr\neq 0$ are:

$v_{19}(x,y,t)=a_0-\mu\left(p+p\tanh\left(\frac{p}{2}\xi\right)\right)$, where $\xi=\lambda\left(x+y+\frac{1+\gamma}{\beta\lambda^2p^2-1}t\right),\mu=\frac{3(1+\gamma)\beta\lambda}{\alpha(\beta\lambda^2p^2-1)}$.

$$v_{20}(x,y,t)=a_0-\mu\left(p+p\coth\left(\frac{p}{2}\xi\right)\right),$$

$$v_{21,22}(x,y,t)=a_0-\mu\left(p+p\left(\tanh(p\xi)\pm i\,\mathrm{sech}(p\xi)\right)\right),$$

$$v_{23,24}(x,y,t)=a_0-\mu\left(p+p\left(\coth(p\xi)\pm \mathrm{csch}(p\xi)\right)\right),$$

$$v_{25}(x,y,t)=a_0+\frac{\mu}{2}\left(2p+p\left(\tanh\left(\frac{p}{4}\xi\right)-\coth\left(\frac{p}{4}\xi\right)\right)\right),$$

$$v_{26}(x,y,t)=a_0+\mu\left(-p+\frac{p\sqrt{(A^2+B^2)}-A\,p\cosh(p\xi)}{A\sinh(p\xi)+B}\right),$$

$$v_{27}(x,y,t)=a_0-\mu\left(-p-\frac{p\sqrt{(B^2-A^2)}+A\,p\sinh(p\xi)}{A\cosh(p\xi)+B}\right).$$

where A and B are two non-zero real constant and satisfies $B^2-A^2>0$.

Family 3: When $r=0$ and $qp\neq 0$ are :

$$v_{30}(x,t)=a_0-\frac{2kp\,\mu}{(k+\cosh(p\,\xi)-\sinh(p\,\xi))},$$

$v_{31}(x,t)=a_0-\frac{2p\mu(\cosh(p\,\xi)+\sinh(p\,\xi))}{(k+\cosh(p\,\xi)+\sinh(p\,\xi))}$, where $\xi=\lambda\left(x+y+\frac{1+\gamma}{\beta\lambda^2p^2-1}t\right)$, $\mu=\frac{3(1+\gamma)\beta\lambda}{\alpha(\beta\lambda^2p^2-1)}$,

Exact traveling wave solution of Eq.(12) for **Case3** given by the following:

Family 1: $p^2-4qr<0$ and $pq\neq 0$ or $qr\neq 0$ are:

$v_{32}(x,y,t)=a_0-\frac{3\beta\omega\lambda}{\alpha}\left(-p+\sqrt{\Delta}\tan\left(\frac{\sqrt{\Delta}}{2}\xi\right)\right)$, where $\xi=\lambda(x+y-\omega t)$, $\Delta=\frac{1+\gamma-\omega}{\beta\omega\lambda^2}$.

$$v_{33}(x,y,t)=a_0+\frac{3\beta\omega\lambda}{\alpha}\left(p+\sqrt{\Delta}\cot\left(\frac{\sqrt{\Delta}}{2}\xi\right)\right),$$

$$v_{34,35}(x,y,t)=a_0-\frac{3\beta\omega\lambda}{\alpha}\left(-p+\sqrt{\Delta}\left(\tan(\sqrt{\Delta}\xi)\pm\sec(\sqrt{\Delta}\xi)\right)\right),$$

$$v_{36,37}(x,y,t)=a_0+\frac{3\beta\omega\lambda}{\alpha}\left(p+\sqrt{\Delta}\left(\cot(\sqrt{\Delta}\xi)\pm\csc(\sqrt{\Delta}\xi)\right)\right),$$

$$v_{38}(x,y,t) = a_0 - \frac{3\beta\omega\lambda}{\alpha}\left(-2p + \sqrt{\Delta}\left(tan\left(\frac{\sqrt{\Delta}}{4}\xi\right) - cot\left(\frac{\sqrt{\Delta}}{4}\xi\right)\right)\right),$$

$$v_{39}(x,y,t) = a_0 - \frac{3\beta\omega\lambda}{\alpha}\left(-p + \frac{\sqrt{\Delta(A^2-B^2)} - A\sqrt{\Delta}\cos(\sqrt{\Delta}\xi)}{A\sin(\sqrt{\Delta}\xi)+B}\right),$$

$$v_{40}(x,y,t) = a_0 - \frac{3\beta\omega}{\alpha}\left(-p - \frac{\sqrt{\Delta(A^2-B^2)} + A\sqrt{\Delta}\cos(\sqrt{\Delta}\xi)}{A\sin(\sqrt{\Delta}\xi)+B}\right),$$

where A and B are two non-zero real constant and satisfies $A^2 - B^2 > 0$.

$$v_{41}(x,y,t) = a_0 - \varphi\left(\frac{-2r\cos\left(\frac{\sqrt{\Delta}}{2}\xi\right)}{\sqrt{\Delta}\sin\left(\frac{\sqrt{\Delta}}{2}\xi\right) + p\cos\left(\frac{\sqrt{\Delta}}{2}\xi\right)}\right),$$

where $\xi = \lambda(x + y - \omega t)$, $\Delta = \frac{1+\gamma-\omega}{\beta\omega\lambda^2}$ $\varphi = \frac{3(1+\gamma-\omega+\beta\omega\lambda^2p^2)}{2\alpha r\lambda}$.

$$v_{42}(x,y,t) = a_0 - \varphi\left(\frac{2r\sin\left(\frac{\sqrt{\Delta}}{2}\xi\right)}{-p\sin\left(\frac{\sqrt{\Delta}}{2}\xi\right) + \sqrt{\Delta}\cos\left(\frac{\sqrt{\Delta}}{2}\xi\right)}\right),$$

$$v_{43,44}(x,y,t) = a_0 - \varphi\left(\frac{-2r\cos\left(\frac{\sqrt{\Delta}}{2}\xi\right)}{\sqrt{\Delta}\sin\left(\frac{\sqrt{\Delta}}{2}\xi\right) + p\cos\left(\frac{\sqrt{\Delta}}{2}\xi\right) \pm \sqrt{\Delta}}\right),$$

$$v_{45,46}(x,y,t) = a_0 - \varphi\left(\frac{2r\sin\left(\frac{\sqrt{\Delta}}{2}\xi\right)}{-p\sin\left(\frac{\sqrt{\Delta}}{2}\xi\right) + \sqrt{\Delta}\cos\left(\frac{\sqrt{\Delta}}{2}\xi\right) \pm \sqrt{\Delta}}\right),$$

$$v_{47}(x,y,t) = a_0 - \varphi\left(\frac{4r\sin\left(\frac{\sqrt{\Delta}}{4}\xi\right)\cos\left(\frac{\sqrt{\Delta}}{4}\xi\right)}{-2p\sin\left(\frac{\sqrt{\Delta}}{4}\xi\right)\cos\left(\frac{\sqrt{\Delta}}{4}\xi\right) + 2\sqrt{\Delta}\cos^2\left(\frac{\sqrt{\Delta}}{4}\xi\right) - \sqrt{\Delta}}\right),$$

Family 2: $p^2 - 4qr > 0$ and $pq \neq 0$ or $qr \neq 0$ are:

we get the same solution as $[v_{32}, \dots, v_{38}], and$

$$v_{48}(x,y,t) = a_0 - \frac{3\beta\omega\lambda}{\alpha}\left(-p + \frac{\sqrt{\varphi(A^2+B^2)} - A\sqrt{\varphi}\cosh(\sqrt{\varphi}\xi)}{A\sinh(\sqrt{\varphi}\xi)+B}\right),$$

$$v_{49}(x,y,t) = a_0 - \frac{3\beta\omega\lambda}{\alpha}\left(-p - \frac{\sqrt{\varphi(B^2-A^2)} + A\sqrt{\varphi}\sinh(\sqrt{\varphi}\xi)}{A\cosh(\sqrt{\varphi}\xi)+B}\right).$$

where $\xi = \lambda(x + y - \omega t)$, $\varphi = -\frac{1+\gamma-\omega}{\beta\omega\lambda^2}$.

where A and B are two non-zero real constant and satisfies $B^2 - A^2 > 0$.

we get the same solutions $[v_{41}, \dots, v_{47}]$.

Conclusion: In this paper, the extended generalized Riccati equation mapping method has been successfully implemented to find new traveling waves solutions for our new proposed equations. The results show that this method is a powerful mathematical tool for solve other nonlinear partial differential equations.

References

[1] A., Akbulut, H., Almusawa, M., Kaplan, and M.S. Osman. On the Conservation Laws and Exact Solutions to the (3+1)-Dimensional Modified KdV-Zakharov-Kuznetsov Equation. Symmetry, 13:5 (2021),765.

[2] Ali,A., Iqbal, M.A., and Mohyud-Din,S.T. Traveling wave solutions of generalized Zakharov-Kuznetsov-Benjamin-Bona-Mahony and simplified modified form of Camassa-Holm equation exp(-φ(η))-Expansion method. Egyptian Journal of Basic and Applied Sciences, 3:2(2016),134-140.

[3] Abdul-Majid. Wazwaz ,Two new painlevė integrable KdV- calogero-Bogoyavlenskii-Schiff (KdV-CBS) equation and new negative-order KdV-CBS equation. Nonlinear Dynamics (2021):1-5.

[4] Akinyemi, Lanre, Mehmet Şenol, and M. S. Osman .Analytical and approximate solutions of nonlinear Schrödinger equation with higher dimension in the anomalous dispersion regime. Journal of Ocean Engineering and Science (2022).

[5] De Bouard, A., and Saut, J.C. Solitary waves of generalize Kadomtsev-Petviashili equations . In Annales de lʹlnstitut Henri Poincarė C, Analyse nonlinėar.

(1997, January),(Vol.14,No. 2, pp. 211-236).Elsevier Masson.

[6] Emad H., Zahran1, Mostafa M., Khater2, " Extended Jacobian Elliptic Function Expansion Method and Its Applications in Biology" Applied Mathematics, (2015), 6, 1174-1181.

[7] H., Naher1 and F., A. Abdullah" The Modified Benjamin Bona-Mahony Equation via the Extended Generalized Riccati Equation Mapping Method"Applied Mathematical Sciences, (2012),Vol.6, no.111,5495–5512.

[8] He X. J., Lü, X., and Li, M. G. Bäcklund transformation, Pfaffian, Wronskian and Grammian solutions to the (3+1)-dimensional generalized Kadomtsev-Petviashvili equation. Anal Math Phys. 2021; 11(1):1. http ://dx.doi .org/ 10 .1007 /s13324-020-00414-y.

[9] Hamad, I. S., and Ali, K. K. Invsetigation of Brownian motion in stochastic Schrödiner wave equation using the modified generalized Riccati equation mapping method. Optical and Quantum Electronics, (2024) 56(6),1-23.

[10] Juan-Juan, J., Ye-Cai, G., Lan-Fang, Z. and Chao-Long, Z. A Table Lookup Method for Exact Analytical Solutions of Nonlinear Fractional Partial Differential Equations, Mathematical Problems in Engineering. 2017(1), 6328438.

[11] Liu, J. G., Zhu, W. H., Osman, M. S., and Ma, W. X. An explicit plethora of different classes of interactive lump solution for an extension form of 3D-Jimbo-Miwa model. Eur. Phys. J. plus 2020,135,412.

[12] Maher, A., El-Hawary, H. M., Al-Amry, M. S. New exact solutions for new model nonlinear partial differential equation, Journal of Applied Mathematics, 2013(1), 767380.

[13] Nasreen, Naila, Aly R. Seadawy, and Dianchen Lu. Construction of soliton solutions for modified Kawahara equation arising in shallow water waves using novel techniques. International Journal of Modern Physics B 34. 07 (2020): 2050045.

[14] Seadawy, Aly R., Mujahid Iqbal, and Dianchen Lu. Applications of propagation of long-wave with dissipation and dispersion in nonlinear media via solitary wave solutions of generalized Kadomtsev-Petviashili modified equal width dynamical equation. Computers and Mathematics with Applictions 78.11 (2019): 3620-3632.

[15] Song, M., and Liu, Z. Periodic wave solutions and their limits for the generalized KP- BBM equation . Journal of Applied Mathematics, (2012), 363879

[16] Wazwaz, A. M. New solitons and kink solutions for the Gardner equation. Communications in nonlinear science and numerical simulation , 12.8(2007): 1395-1404.

[17] Wazwaz, A. M. A new integrable equation that combines the kdv equation with the negative order kdv equation. Mathematical Methods in the Applied Sciences, 41.1(2018):80-87.

[18] Wazwaz, A. M. A new integrable equation constructed via combining the recursion operator of the Calogero –Bogoyavlenskii-Schiff (CBS) equation and its inverse operator. Appl. Math . Inf. Sci, 11.5(2017):1241-1246 .

[19] Wang, Mingliang., Exact solutions for a compound KdV-Burgers equation, Physics Letters A, (1996), 213(5-6):279-287.

[20] Zhu, S.D., The generalizing Riccati equation mapping method in non-linear evolution equation: application to (2+1)-dimensional Boiti–Leon–Pempinelle equation, Chaos, Solitons & Fractals. (2008), Sep 1;37(5):1335-42.

الحلول الدقيقة للمعادلات التفاضلية الجزئية غير الخطية الجديدة المحتملة باستخدام طريقة رسم معادلة ريكاتي المعممة الممتدة

مبارك عبود حميد القفيل[1]
قسم الرياضيات كلية التربية عدن، جامعة عدن

سلمى عبدالله الدعمكي[2]
قسم الرياضيات كلية التربية عدن، جامعة عدن

الملخص: في هذه الورقة، نقدم نماذج جديداً لمعادلة كادومتسيف بيتفياشفيلي وبنيامين بونا ماهوني، وبالتحديد معادلة محتملة لمعادلة (KP- BBM) والمعادلة المحتملة من دمج " كورتويج دي فريس وكورتويج دي فريس ذات الترتيب السالب" و"معادلة كالوجيرو – بوجويافلينسكي شيف وكالوجيرو – بوجويافلينسكي شيف ذات الترتيب السالب "، وهي معادلة محتملة لمعادلة (KdV-nKdV و c(CBS- nCBS. ونطبق طريقة رسم معادلة ريكاتي المعممة الممتدة لحل النماذج الجديدة.

نحصل على حلول دقيقة للموجات المتنقلة ونعبر عنها بالدوال الزائدية والدوال المثلثية والدوال الكسرية.

الكلمات المفتاحية: معادلة محتملة لمعادلة (KP-BBM)، معادلة محتملة لمعادلة c(KdV-nKdV and CBS-nCBS) ، حلول دقيقة وطريقة رسم معادلة ريكاتي المعممة الممتدة.