Comparison Between Two Methods to Find Exact Solutions for a New Models of Nonlinear Partial Differential Equations

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Abstract: The aim of this paper is to give a connection between two methods, the $\left(\frac{G'}{G^2}\right)$-expansion method and the tanh method which are used to find exact solutions of nonlinear partial differential equations (NLPDEs). It has been shown that these two methods are the same in special conditions. For illustration we present a new models of Equal-Width (EW) equation and Vaknenko-Parks (VP) equation. Exact traveling wave solutions are obtained and expressed in terms of hyperbolic functions, trigonometric functions and rational functions, with the aid of the software Maple.

Keywords: Equal-Width (EW)equation, Vaknenko-Parkes (VP) equation, modified Vaknenko-Parks (mVP) equation, exact solutions, the $\left(\frac{G'}{G^2}\right)$-expansion method and the tanh method.

1.Introduction: Many physical phenomena can be represented by nonlinear partial differential equations (NLPDEs). Therefore, the investigation of exact traveling wave solutions of nonlinear partial differential equations has become of great interest to the researches. Many methods have been developed for obtaining exact solutions of NLPDEs such as: the generalized exponential rational method [1], the extended tanh function method [2], the first integer method [3], a new rational Sine-Goron method [4], the Jacobi elliptic function expansion method [5], the generalized Riccati equation mapping method [6], the $\left(\frac{G'}{G},\frac{G'}{G^2}\right)$-expansion method [7], the extended mapping method [8], the modification of fan sub-equation method [9] and the reduction mKdV method [10].

The tanh method is proposed by Malfiet [11], this method is used for the computation of exact traveling wave solutions [12,13]. The second method is the $\left(\frac{G'}{G^2}\right)$- expansion method has been proposed by Li Wen-An, Chen Hao and Zhang Guo-Cai [14], this method is an interesting method to obtain exact solutions for nonlinear partial differential equations [15,16]. In this paper we prove that the tanh method is a special case of $\left(\frac{G'}{G^2}\right)$-expansion method.

2. Description of the methods

2.1. The $\left(\frac{G'}{G^2}\right)$- expansion method

Suppose we have the following nonlinear partial differential equation

$$\mathcal{N}(u, u_x, u_t, u_{xx}, u_{xt}, \dots) = 0, \quad (1)$$

where $u = u(x,t)$ is an unknown function, $\mathcal{N}$ is nonlinear of its arguments, the subscript denotes the partial derivatives.

Step 1. Use the wave variable $\xi = \alpha(x - \beta t)$, where α and β are the wave number and the wave speed respectively, to change the NLPDE into NLODE.

$$Q(u, u', u'', \dots) = 0, \quad ' \equiv \frac{d}{d\xi}. \tag{2}$$

Step 2. Suppose the traveling wave solution of Eq. (2), can be expresses in the form

$$u(x,t) = u(\xi) = a_0 + \sum_{i=1}^{m} a_i \left(\frac{G'}{G^2}\right)^i, \tag{3}$$

where the coefficients a_0, a_i, α and β are constants to be determined later and $\left(\frac{G'}{G^2}\right)$ satisfies a nonlinear ordinary differential equation,

$$\left(\frac{G'}{G^2}\right)' = \mu + \lambda \left(\frac{G'}{G^2}\right)^2, \tag{4}$$

where μ and λ are arbitrary constants, shut that $\mu \neq 1$ and $\lambda \neq 0$. The parameter m is a positive integer and can be determined via balancing between the highest order derivative terms and the nonlinear term in Eq. (2).

Step 3. Substituting Eq. (3) into Eq. (2) and using Eq. (4). Collecting all terms of the same power order of $\left(\frac{G'}{G^2}\right)^i, (i = 0,1,2, \dots, m)$. Setting each coefficient to zero yields a set of algebraic equations which can be solved and obtained all the constants a_0, $a_i(i = 1, 2, \dots, m), \alpha$ and β with the aid of Maple.

The ODE (4) has the following solutions

Case 1: If $\lambda\mu > 0$, then

$$\left(\frac{G'}{G^2}\right) = \sqrt{\frac{\mu}{\lambda}} \left(\frac{C\cos(\sqrt{\lambda\mu}\,\xi) + D\sin(\sqrt{\lambda\mu}\,\xi)}{D\cos(\sqrt{\lambda\mu}\,\xi) - C\sin(\sqrt{\lambda\mu}\,\xi)}\right), \tag{5}$$

Case 2: If $\lambda\mu < 0$, then

$$\left(\frac{G'}{G^2}\right) = -\frac{\sqrt{|\lambda\mu|}}{\lambda} \left(\frac{C\cosh\left(2\sqrt{|\lambda\mu|}\,\xi\right) + C\sinh\left(\sqrt{|\lambda\mu|}\,\xi\right) + D}{C\cosh\left(2\sqrt{|\lambda\mu|}\,\xi\right) + C\sinh\left(\sqrt{|\lambda\mu|}\,\xi\right) - D}\right), \tag{6}$$

Case 3: If $\mu = 0, \lambda \neq 0$, then

$$\left(\frac{G'}{G^2}\right) = \frac{-C}{\lambda(C\xi + D)}. \tag{7}$$

In the above expressions, C and D are nonzero constants.

Step 4. Substituting the obtain constants and general solution of Eq. (4) into Eq. (3), we have the traveling wave solutions of nonlinear partial differential equation (1).

2.2 The tanh method: The tanh method for Eq. (1), is given by

Step 1. Use the wave variable $\xi = \alpha(x - \beta t)$ to change the NLPDE into NLODE.

Step 2. The tanh method admits the use of finite expansion

$$u(\eta\xi) = S(Y) = \sum_{i=0}^{m} b_i Y^i = \sum_{i=0}^{m} b_i \tanh^i(\eta\xi), \tag{8}$$

$$Y' = \eta(1 - Y^2). \tag{9}$$

The parameter m is a positive integer and can be determined via balancing between the highest order derivative terms and the nonlinear term in Eq. (2).

Step 3. Substituting Eq. (8) into Eq. (2) and using Eq. (9), yields an algebraic equation in power of Y. Collect all coefficients of power $Y^i, i = (0,1,2, \dots, m)$ in resulting equation where these coefficients have to vanish. This will give a system of algebraic equations involving the parameters $b_i(i = 0,1,2, \dots, m)$, α and β.

With the aid of the computer program Maple, we can solve the set of nonlinear algebraic equations and obtain all the constants $b_i(i = 1, 2, \dots, m), \alpha$ and β.

Step 4. Having determined these constants and using Eq. (8), we have the traveling wave solutions of nonlinear partial differential equation (1).

3. Connection between the $\left(\frac{G'}{G^2}\right)$- expansion method and the tanh method

The tanh method is a special case of the $\left(\frac{G'}{G^2}\right)$- expansion method.

Proof:

Using the general solution of equation $Y = \tanh(\eta\xi)$, we have: for $\lambda\mu > 0$,

$\left(\frac{G'}{G^2}\right) = \sqrt{\frac{\mu}{\lambda}}\left(\frac{C\cos(\sqrt{\lambda\mu}\,\xi)+D\sin(\sqrt{\lambda\mu}\,\xi)}{D\cos(\sqrt{\lambda\mu}\,\xi)-C\sin(\sqrt{\lambda\mu}\,\xi)}\right)$, where C, D, μ and λ are arbitrary constants.

If we take $C = 0, \lambda = 1$ in Eq. (5), we get the following form

$$\frac{G'}{G^2} = \sqrt{\mu}\, tan\, \sqrt{\mu}\,\xi, \tag{10}$$

since $\lambda\mu > 0, \lambda = 1$. Suppose that $\sqrt{\mu} = \sqrt{-\eta}$, by substituting Eq. (10) turns to

$$\frac{G'}{G^2} = \sqrt{-\eta}\tan\sqrt{-\eta}\,\xi,$$

$$\frac{G'}{G^2} = -\sqrt{\eta}\, tanh\, \sqrt{\eta}\,\xi\, . \tag{11}$$

Substituting Eq. (11) into Eq. (3), we get

$$u(\xi) = \sum_{i=0}^{m} a_i\, \left(-\sqrt{\eta}\tanh\sqrt{\eta}\,\xi\right)^i,$$

$$u(\xi) = \sum_{i=0}^{m} b_i\, tanh^i(\delta\xi)\,, \quad \delta = \sqrt{\eta}\,, \quad b_i = -a_i\delta^i, \quad i = 0,1,2,\ldots,m. \tag{12}$$

By comparing Eq. (8) and Eq. (12), we conclude that the result of the tanh method can be obtained directly by the $\left(\frac{G'}{G^2}\right)$- expansion method under the special conditions $C = 0$ and $\lambda = 1$.

4. Applications

4.1 Exact solutions for potential EW equation

4.1.1 Using $\left(\frac{G'}{G^2}\right)$- expansion method: In this section, we solve our new equation, namely a potential EW equation as the form

$$u_t + 2\theta(u_x)^2 - \omega u_{xxt} = 0, \quad u = u(x,t), \quad \theta, \omega \in R, \tag{13}$$

and denoted by potential Equal-Width (pEW) equation, where

$$u_t + 2\theta u u_x - \omega u_{xxt} = 0, \quad u = u(x,t), \quad \theta, \omega \in R\,, \tag{14}$$

is the Equal-Width (EW) equation [17].

We get the original solutions for our new equation as the following

Substituting $\xi = \alpha(x - \beta t)$ in Eq. (13), we obtain

$$-\beta u' + 2\theta\alpha(u')^2 + \omega\alpha^2\beta u''' = 0\,, \tag{15}$$

Balancing the order of the nonlinear term $(u')^2$ with the highest derivative u''' gives $2(m+1) = m+3$ that gives $m = 1$.

The solution of Eq. (15) has the form

$$u(\xi) = a_0 + a_1\left(\frac{G'}{G^2}\right), \tag{16}$$

Substituting Eq. (16) in Eq. (15) and using Eq. (4), collecting the coefficients of each power of $\left(\frac{G'}{G^2}\right)^i, 0 \le i \le 4$, setting each coefficient to zero, and solving the resulting system with the aid of Maple, we obtain the following sets of solutions

1. $a_1 = 0, \alpha = \alpha, \beta = \beta,$
2. $a_1 = -\frac{3\beta\lambda\omega}{\theta}\sqrt{-\frac{1}{4\lambda\mu\omega}}, \alpha = \sqrt{-\frac{1}{4\lambda\mu\omega}}, \beta = \beta,$

3. $a_1 = \frac{3\beta\lambda\omega}{\theta}\sqrt{-\frac{1}{4\lambda\mu\omega}}, \alpha = -\sqrt{-\frac{1}{4\lambda\mu\omega}}, \beta = \beta.$

Using Eq. (16), the solution of Eq. (4), and the above sets of solutions [1-3], we get

$u_1(x,t) = 0,$ trivial solution

For $\lambda\mu > 0$, we obtain

$$u_{2,3} = \mp\frac{3}{2}i\frac{\beta\omega\sqrt{\frac{1}{\omega}}\left(C\cosh\left(\frac{\xi}{2}\right)\pm iD\sinh\left(\frac{\xi}{2}\right)\right)}{\theta\left(D\cosh\left(\frac{\xi}{2}\right)\mp iC\sinh\left(\frac{\xi}{2}\right)\right)},$$

where C, D, ω, θ are arbitrary constant and $\xi = \sqrt{\frac{1}{\omega}}(x - \beta t).$

For $\lambda\mu < 0$, we obtain

$$u_{4,5} = \pm\frac{3}{2}\frac{\omega\sqrt{\frac{1}{\omega}}(C\cosh(\xi)\pm C\sinh(\xi)+D)}{\theta(C\cosh(\xi)\pm C\sinh(\xi)-D)}.$$

Fig (1) Graph of singular periodic solution $u_2(x,t)$ when

$\omega = -1, \beta = 2, C = 1, D = 2, \theta = 2$

Fig (2) Graph of singular periodic solution $u_3(x,t)$ when

$\omega = -1, \beta = 1, C = 2, D = 1, \theta = -1$

Fig (3) Graph of kink solution $u_4(x,t)$ when

$\omega = 1, \beta = -0.5, C = 1, D = -1, \theta = 1$

4.1.2 Using the tanh method: By using the above technique in (4.1.1), the solution of Eq. (15) has the form

$$u = b_0 + b_1 Y,\quad Y = tanh(\eta\xi). \tag{17}$$

Substituting Eq. (17) in Eq. (15) and using Eq. (9). Collecting of each power of Y^i, $0 \le i \le 4$, we obtain the following sets of solutions

1. $b_1 = 0, \alpha = \alpha, \beta = \beta,$

2. $b_1 = \frac{3\beta\omega\sqrt{\frac{1}{\omega}}}{2\theta}, \alpha = \frac{1}{2\eta}\sqrt{\frac{1}{\omega}}, \beta = \beta,$

3. $b_1 = -\frac{3\beta\omega\sqrt{\frac{1}{\omega}}}{2\theta}, \alpha = -\frac{1}{2\eta}\sqrt{\frac{1}{\omega}}, \beta = \beta.$

The solution of Eq. (15)

$u_1(x,t) = 0,$ trivial solution

$u_2(x,t) = \frac{3\beta\omega\sqrt{\frac{1}{\omega}}}{2\theta}\tanh\left(\frac{\xi}{2}\right)$, where ω, θ are arbitrary constants and $\xi = \sqrt{\frac{1}{\omega}}(x - \beta t).$

Fig (4) Graph of kink solution $u_2(x,t)$ when

$\omega = 1, \beta = 2, \theta = 1$

The solution

$u_2 = -\frac{3}{2} i \frac{\beta\omega\sqrt{\frac{1}{\omega}}\left(C\cosh\left(\frac{\xi}{2}\right) \pm iD\sinh\left(\frac{\xi}{2}\right)\right)}{\theta\left(D\cosh\left(\frac{\xi}{2}\right) \mp iC\sinh\left(\frac{\xi}{2}\right)\right)}$ by using the $\left(\frac{G'}{G^2}\right)$- expansion method is, the same solution

$u_2(x,t) = \frac{3\beta\omega\sqrt{\frac{1}{\omega}}}{2\theta}\tanh\left(\frac{\xi}{2}\right)$ by using the tanh method if we put $C = 0$.

4.2 Exact solution for cmVP equation

4.2.1 Using $\left(\frac{G'}{G^2}\right)$- expansion method: In this section, we solve our new equation, namely a combined Vaknenko-Parkes and the modified Vaknenko-Parkes equation as the form

$$vv_{xxt} - v_x v_{xt} + (v^2 + v^3)v_t = 0, \quad v = v(x,t), \tag{18}$$

and donated by (cmVP), where

$vv_{xxt} - v_x v_{xt} + v^2 v_t = 0,$

is the Vaknenko-Parkes (VP) equation [18].

$vv_{xxt} - v_x v_{xt} + v^3 v_t = 0,$

is the modified Vaknenko-Parkes (mVP) equation [19].

Substituting $\xi = \alpha(x - \beta t)$, in Eq. (18), we find

$$\alpha^2 (v')^2 - \alpha^2 v v'' - \frac{v^3}{3} - \frac{v^4}{4} = 0, \tag{19}$$

We get the original solutions for our new equation as the following

Balancing the order of the nonlinear term $(u')^2$ and u^4 gives $m = 1$

The solution of Eq. (18) has the form

$$v(\xi) = a_0 + a_1\left(\frac{G'}{G^2}\right), \tag{20}$$

Substituting Eq. (20) in Eq. (19) and using Eq. (4), collecting the coefficients of each power $\left(\frac{G'}{G^2}\right)^i$,$0 \le i \le 4$**,** setting each coefficient to zero, and solving the resulting system with the aid of Maple, we obtain the following sets of solutions

1. $a_0 = \frac{-4}{3}, a_1 = 0, \alpha = \alpha,$
2. $a_0 = \frac{-2}{3}, a_1 = \frac{2}{3}\sqrt{\frac{-\lambda}{\mu}}, \alpha = \pm\frac{1}{3}\sqrt{\frac{1}{\mu\lambda}},$
3. $a_0 = \frac{-2}{3}, a_1 = -\frac{2}{3}\sqrt{\frac{-\lambda}{\mu}}, \alpha = \pm\frac{1}{3}\sqrt{\frac{1}{\mu\lambda}}.$

Using Eq. (20), the solution of Eq. (4), and the above sets of solutions [1-3], we get

$v_1(x,t) = \frac{-4}{3},$ trivial solution

For $\lambda\mu > 0$, we obtain

$$v_{2,3}(x,t) = -\frac{2}{3} \pm \frac{2}{3} i \left(\frac{C\cos(\sqrt{\lambda\mu}\xi) \pm D\sin(\sqrt{\lambda\mu}\xi)}{D\cos(\sqrt{\lambda\mu}\xi) \mp C\sin(\sqrt{\lambda\mu}\xi)}\right),$$

where C, D are arbitrary constants and $\xi = \frac{1}{3}(x - \beta t)$.

For $\lambda\mu < 0$, we obtain

$$v_{4,5}(x,t) = -\frac{2}{3} \pm \frac{2}{3}\left(\frac{C\cosh\left(2\sqrt{|\lambda\mu|}\xi\right) \pm C\,sinh\left(2\sqrt{|\lambda\mu|}\xi\right) + D}{C\cosh\left(2\sqrt{|\lambda\mu|}\xi\right) \mp C\,sinh\left(2\sqrt{|\lambda\mu|}\xi\right) - D}\right).$$

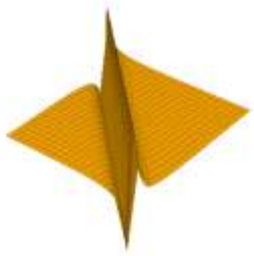

Fig (5) Graph of anti- kink solution $v_4(x,t)$ when

$\lambda=1,\mu=-1,\beta=1,C=2,D=1$

Fig (6) Graph of of anti-kink solution $v_5(x,t)$ when

$\lambda=1,\mu=-1,\beta=-2,C=-3,D=-2$

4.2.2 Using the tanh method: By using the above technique in (4.2.1), the solution of Eq. (19) has the form

$$v=b_0+b_1Y,\ \ Y=tanh(\eta\xi). \tag{21}$$

Substituting Eq. (21) in Eq. (19) and using Eq. (9). Collecting of each power of Y^i, $0\le i\le 4$, setting each coefficient to zero, and solving the resulting system with the aid of Maple, we obtain the following sets of solutions

1. $b_0=-\frac{2}{3}, b_1=\frac{2}{3}, \alpha=\pm\ \frac{i}{3\eta}$

2. $b_0=-\frac{2}{3}, b_1=-\frac{2}{3}, \alpha=\pm\ \frac{i}{3\eta}$

The above set of values yields the following exact solutions for cmVP

$v(x,t)=-\frac{2}{3}+\frac{2}{3}i\tan(\xi)$, where $\xi=\frac{1}{3}(x-\beta t)$.

The solutions

$v_{2,3}(x,t)=-\frac{2}{3}+\frac{2}{3}i\left(\frac{C\cos(\sqrt{\lambda\mu}\xi)\pm D\sin(\sqrt{\lambda\mu}\xi)}{D\cos(\sqrt{\lambda\mu}\xi)\mp C\sin(\sqrt{\lambda\mu}\xi)}\right)$ by using the $\left(\frac{G'}{G^2}\right)$- expansion method is, the same solution

$v(x,t)=-\frac{2}{3}+\frac{2}{3}i\tan(\xi)$ by using the tanh method if we put $C=0,\lambda=1$.

5. Conclusion:

In this paper, we have been looking for a comparison between the $\left(\frac{G'}{G^2}\right)$-expansion method and tanh method. we have analyzed the two new form of Equal-Width (EW) equation and Vakhnenko-Parkes (VP) equation. Exact traveling wave solutions are constructed including periodic wave solutions and kink wave solutions. Many solutions represent graphically with the aid of Scientific WorkPlace by choosing the suitable values of involved parameters. We have succeeded in identifying the equivalence of two methods under special conditions. We proved that the tanh method is a special case of $\left(\frac{G'}{G^2}\right)$- expansion method. It is also a promising methods to solve other nonlinear partial differential equations.

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مقارنة بين طريقتين لإيجاد الحلول الدقيقة لنماذج جديدة من المعادلات التفاضلية الجزئية غير الخطية

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الملخص: الهدف من هذا البحث هو تقديم ارتباط بين طريقتين، طريقة - $\left(\frac{G'}{G^2}\right)$الموسعّة وطريقة tanh، اللتين تُستخدمان لإيجاد الحلول الدقيقة للمعادلات التفاضلية الجزئية غير الخطية (NLPDEs)، وقد توصلنا أن هاتين الطريقتين متساويتان تحت شروط معينة. للتوضيح، نقدم نماذج جديدة لمعادلة العرض المتساوي (EW) ومعادلة فاكينكو- باركس (VP)، تم الحصول على حلول دقيقة لموجات متنقلة والتعبير عنها بواسطة الدوال الزائدية، الدوال المثلثية والدوال الكسرية، بمساعدة برنامج Maple .

الكلمات المفتاحية: معادلة العرض المتساوي(EW) ، معادلة فاكنينكو- باركس(VP) ، معادلة فاكنينكو- باركس المعدّلة (mVP)، الحلول الدقيقة، طريقة $\left(\frac{G'}{G^2}\right)$ – الموسعّة وطريقة tanh .